SU(N) Magnetism with Cold Atoms and Chiral Spin Liquids

Victor Gurarie

collaboration with
M. Hermele, A.M. Rey

Cologne, May 2010

In this talk

- Alkaline earth atoms can be though of as having SU(N) spins, generalization of the usual SU(2) spins.

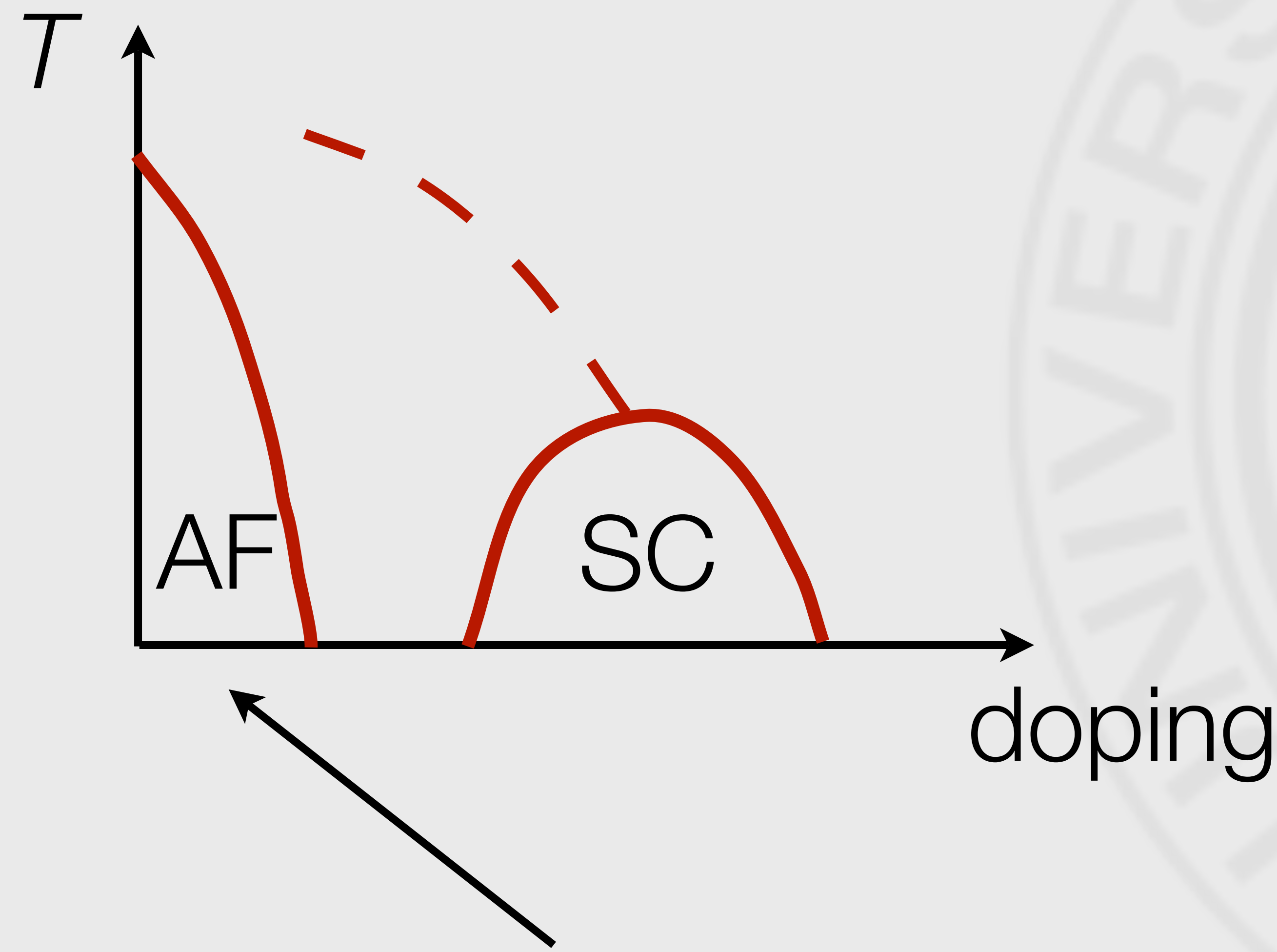
- SU(N) magnets are more controllable theoretically than their SU(2) counterparts and have richer phase diagrams

- "Heisenberg antiferromagnets" of the SU(N) spins can be **Chiral Spin Liquids**, spins counterparts of **quantum Hall effect**, states of matter having excitations with **fractional and non-Abelian statistics.** Those, as is well known, can be used for quantum computation.

SU(N) magnetism

Quantum magnetism

Cuprates’ phase diagram



Antiferromagnet
(occurs when one has close
to one electron per lattice site)

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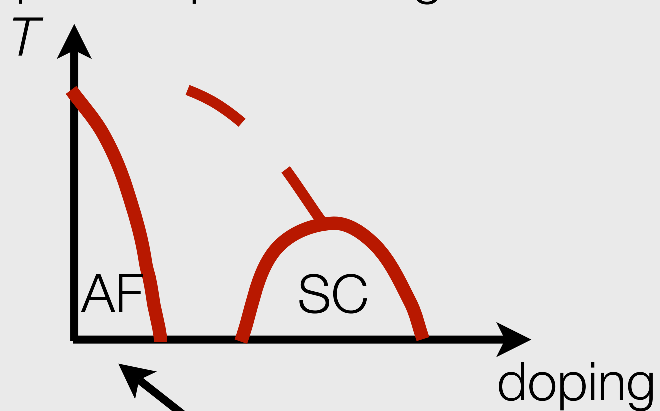

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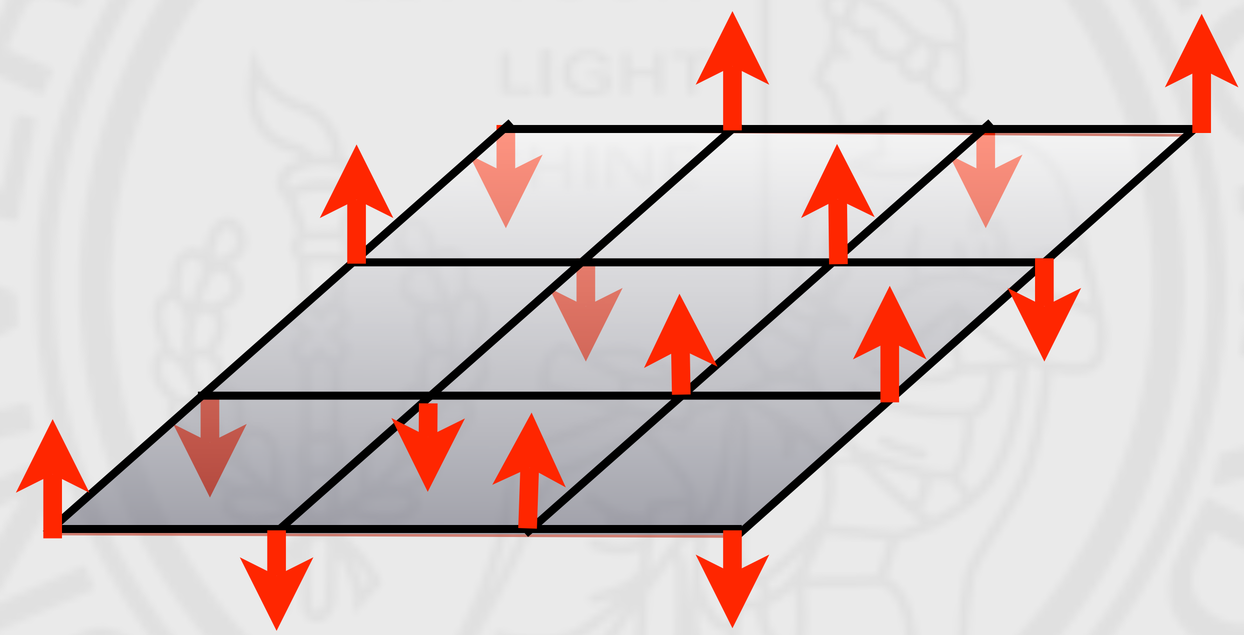
Origin of the antiferromagnetism: Mott insulator of spinful electrons



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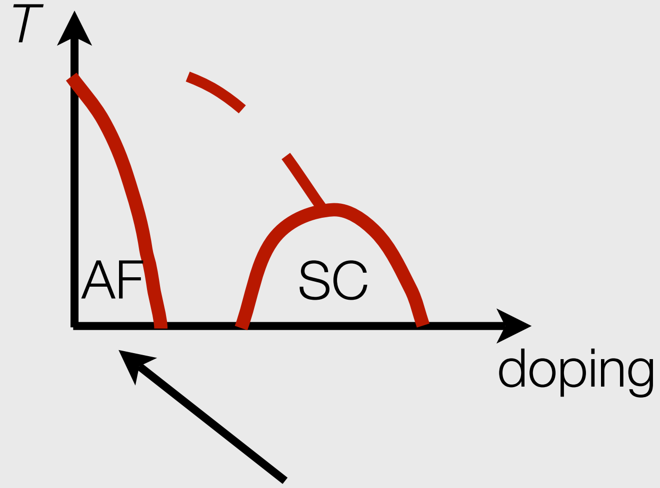

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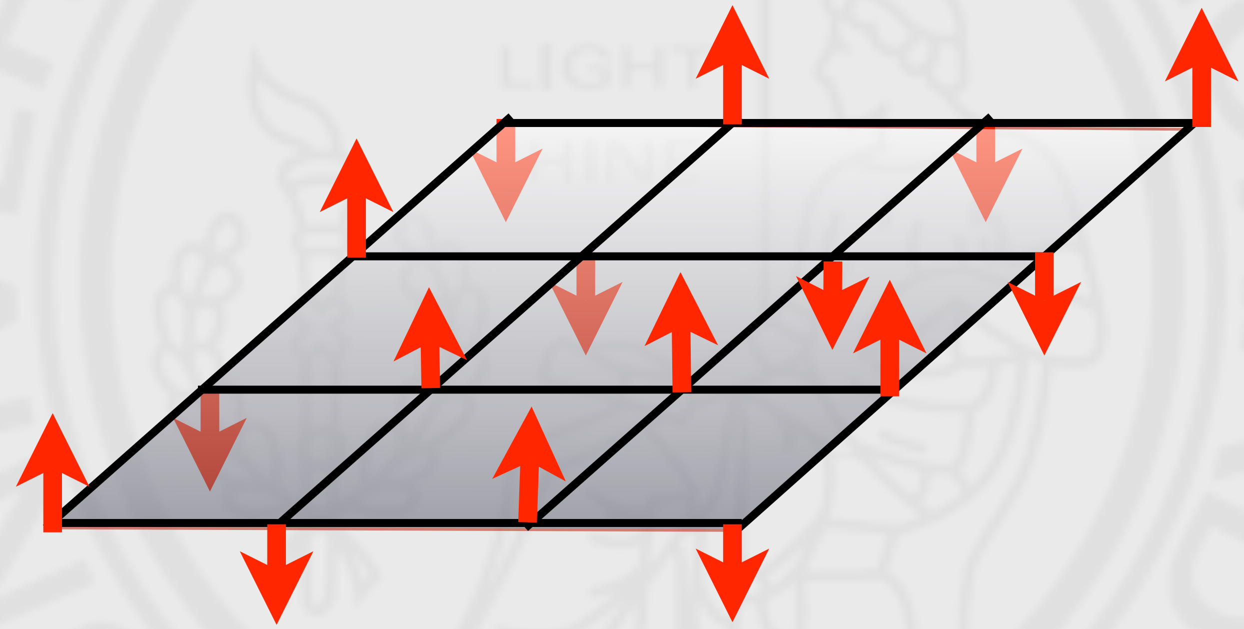

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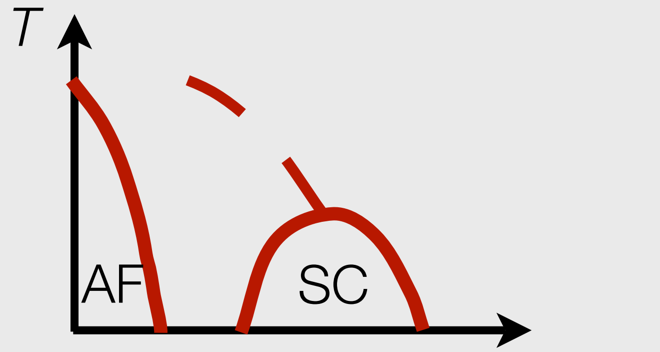

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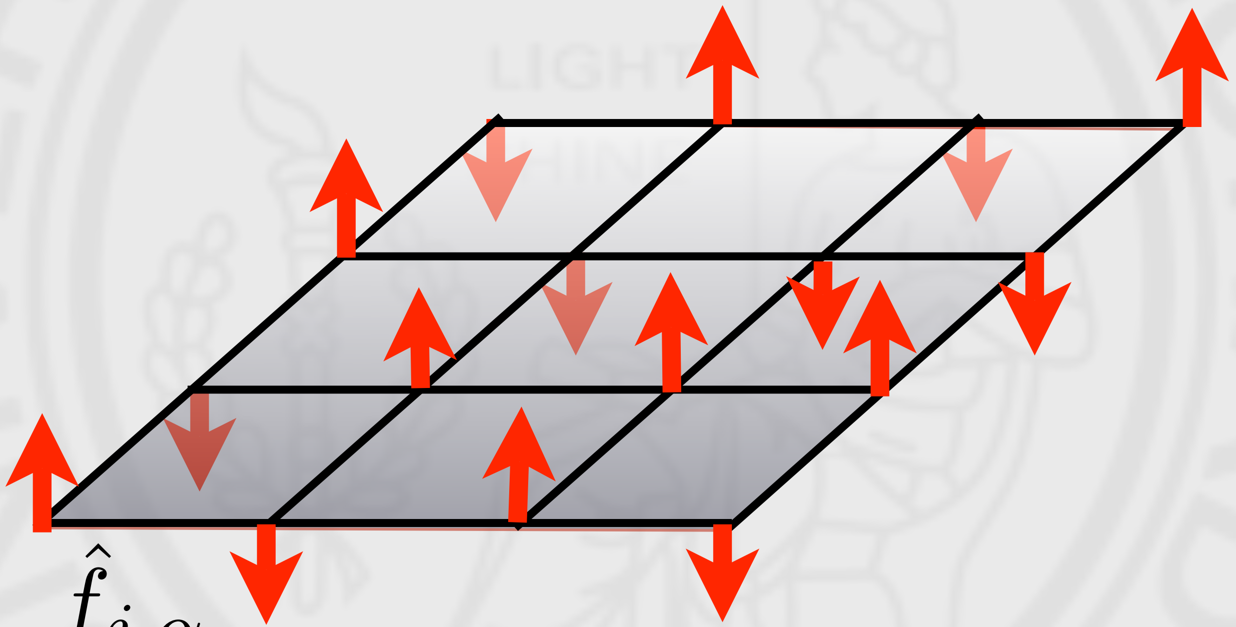

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spin-up spin-down

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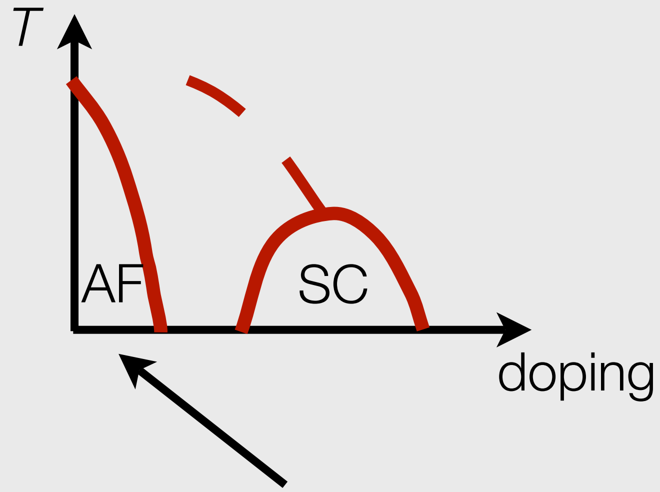

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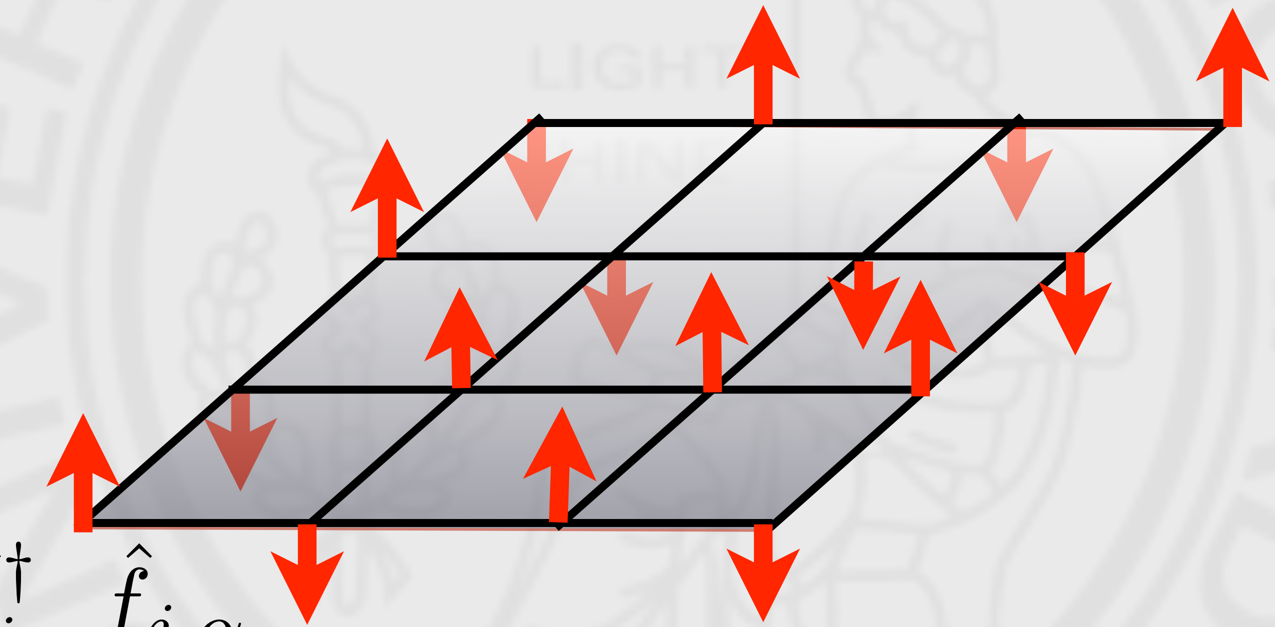

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Spin operators

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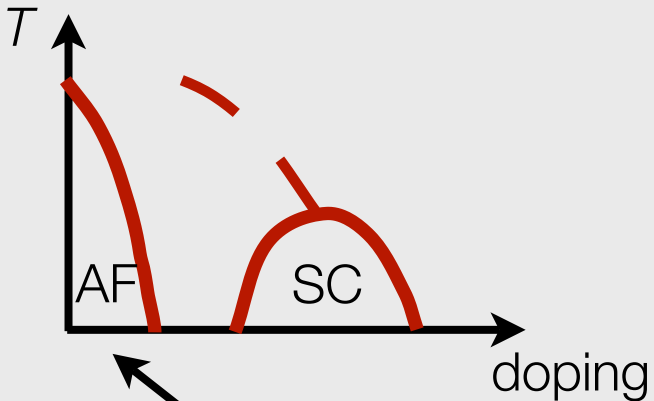

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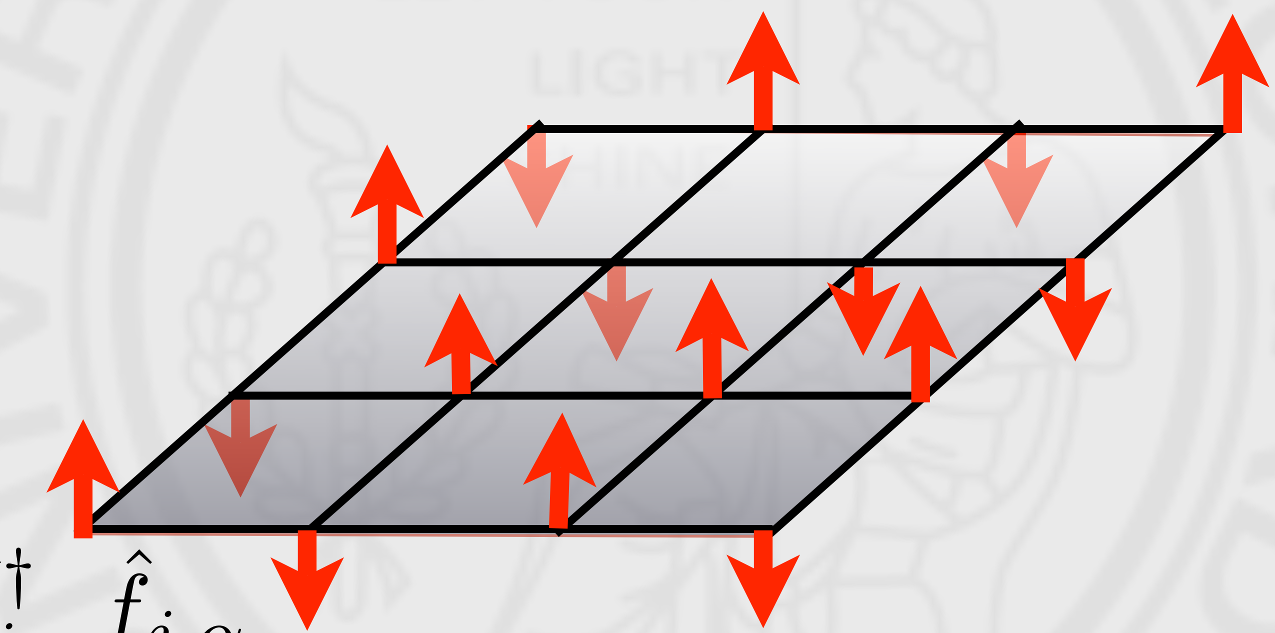

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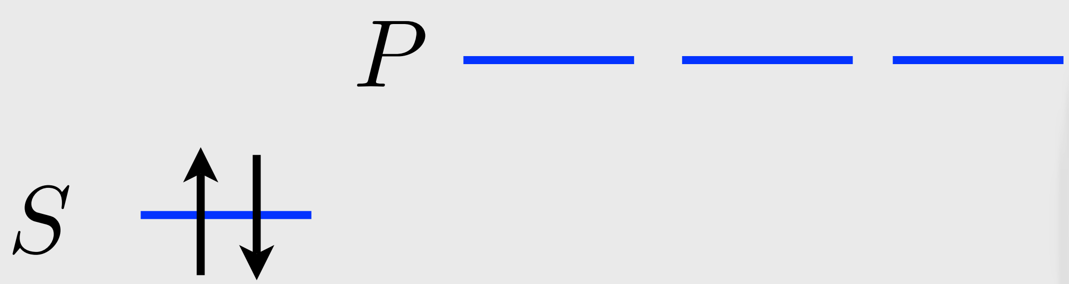
Heisenberg antiferromagnet

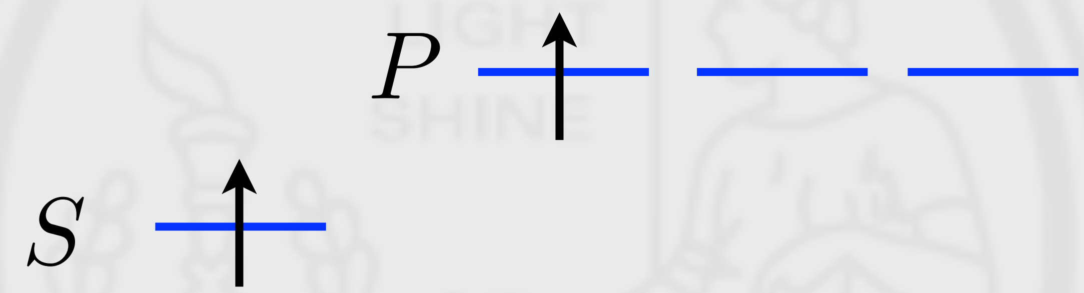
Alkaline earth atoms: SU(N) spins

two electrons in the outer shell

Ground state $^1S_0$



Excited state $^3P_0$



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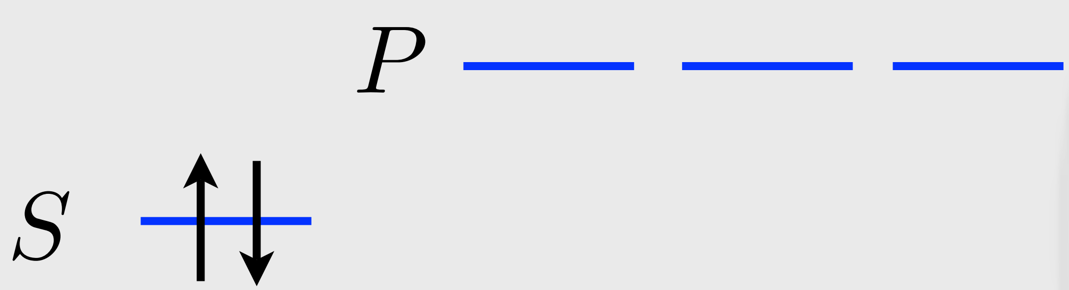

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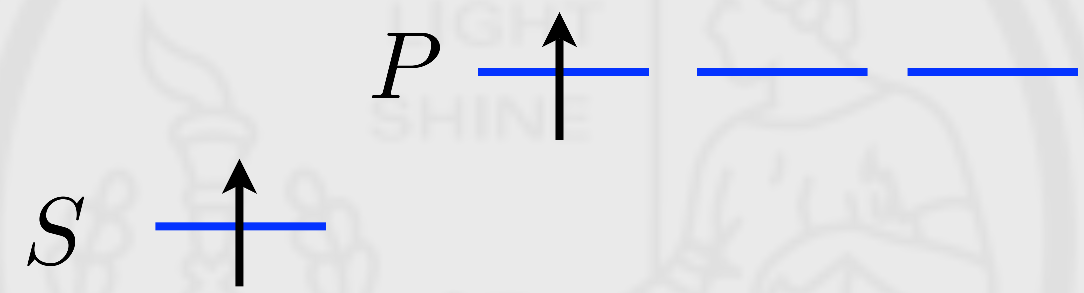

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This (sometimes large) nuclear spin can play the role of the **SU(N) spin**.

For example, **^{87}Sr: I=9/2, N= 2I+1=10**.

A.V. Gorshkov, M. Hermele, VG, C. Xu, P.S. Julienne, J. Ye, P. Zoller, E. Demler, M.D. Lukin, A.M. Rey. (2010)

M. Cazallila, A. Ho, M. Ueda (2009)

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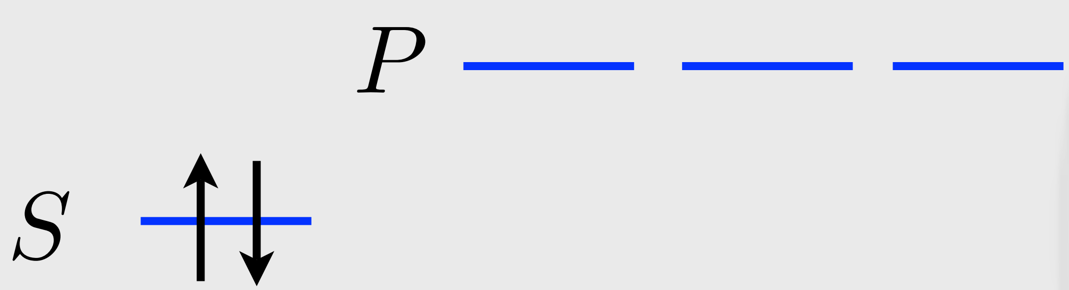

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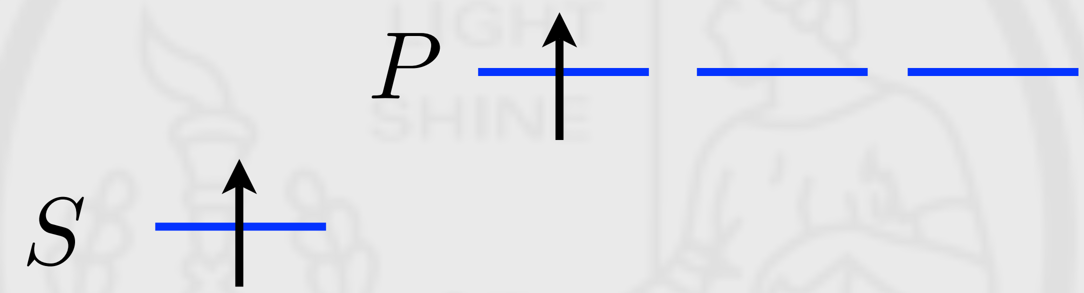

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Interesting twist: only fermionic atoms have N>1 (bosons have even-even nuclei, whose I=0)

Experiments

Selected for a Viewpoint in *Physics*
PRL **103,** 200401 (2009) PHYSICAL REVIEW LETTERS week ending 13 NOVEMBER 2009

Bose-Einstein Condensation of Strontium

Simon Stellmer,[1,2] Meng Khoon Tey,[1] Bo Huang,[1,2] Rudolf Grimm,[1,2] and Florian Schreck[1]
[1]*Institut für Quantenoptik und Quanteninformation (IQOQI), Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
[2]*Institut für Experimentalphysik und Zentrum für Quantenphysik, Universität Innsbruck, 6020 Innsbruck, Austria*
(Received 5 October 2009; published 9 November 2009)

We report on the attainment of Bose-Einstein condensation with ultracold strontium atoms. We use the ^{84}Sr isotope, which has a low natural abundance but offers excellent scattering properties for evaporative cooling. Accumulation in a metastable state using a magnetic-trap, narrowline cooling, and straightforward evaporative cooling in an optical trap lead to pure condensates containing 1.5×10^5 atoms. This puts ^{84}Sr in a prime position for future experiments on quantum-degenerate gases involving atomic two-electron systems.

Selected for a Viewpoint in *Physics*
PRL **103,** 200402 (2009) PHYSICAL REVIEW LETTERS week ending 13 NOVEMBER 2009

Bose-Einstein Condensation of ^{84}Sr

Y. N. Martinez de Escobar, P. G. Mickelson, M. Yan, B. J. DeSalvo, S. B. Nagel, and T. C. Killian
Rice University, Department of Physics and Astronomy, Houston, Texas 77251, USA
(Received 15 October 2009; published 9 November 2009)

We report Bose-Einstein condensation of ^{84}Sr in an optical dipole trap. Efficient laser cooling on the narrow intercombination line and an ideal s-wave scattering length allow the creation of large condensates ($N_0 \sim 3 \times 10^5$) even though the natural abundance of this isotope is only 0.6%. Condensation is heralded by the emergence of a low-velocity component in time-of-flight images.

These are bosons, they have spin 0. We need fermions.

Experiments

Degenerate Fermi Gas of ^{87}Sr

B. J. DeSalvo, M. Yan, P. G. Mickelson, Y. N. Martinez de Escobar, and T. C. Killian
Rice University, Department of Physics and Astronomy, Houston, Texas, 77251
(Dated: May 6, 2010)

We report quantum degeneracy in a gas of ultra-cold fermionic ^{87}Sr atoms. By evaporatively cooling a mixture of spin states in an optical dipole trap for 10.5 s, we obtain samples well into the degenerate regime with $T/T_F = 0.26^{+.05}_{-.06}$. The main signature of degeneracy is a change in the momentum distribution as measured by time-of-flight imaging, and we also observe a decrease in evaporation efficiency below $T/T_F \sim 0.5$.

Realization of SU(2)×SU(6) Fermi System

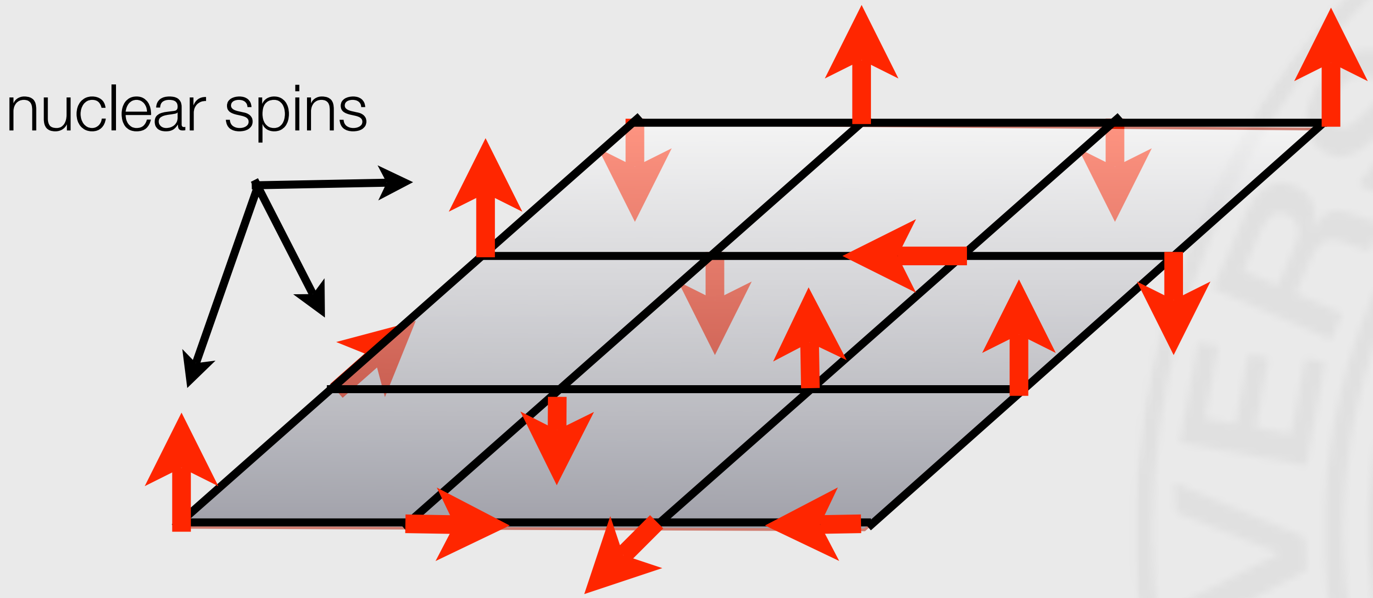
Shintaro Taie,[1,*] Yosuke Takasu,[1] Seiji Sugawa,[1] Rekishu Yamazaki,[1,2] Takuya Tsujimoto,[1] Ryo Murakami,[1] and Yoshiro Takahashi[1,2]
[1] *Department of Physics, Graduate School of Science, Kyoto University, Japan 606-8502*
[2] *CREST, JST, 4-1-8 Honcho Kawaguchi, Saitama 332-0012, Japan*
(Dated: May 21, 2010)

We report the realization of a novel degenerate Fermi mixture with an SU(2)×SU(6) symmetry in a cold atomic gas. We successfully cool the mixture of the two fermionic isotopes of ytterbium ^{171}Yb with the nuclear spin $I = 1/2$ and ^{173}Yb with $I = 5/2$ below the Fermi temperature T_{F} as $0.46T_{\mathrm{F}}$ for ^{171}Yb and $0.54T_{\mathrm{F}}$ for ^{173}Yb. The same scattering lengths for different spin components make this mixture featured with the novel SU(2)×SU(6) symmetry. The nuclear spin components are separately imaged by exploiting an optical Stern-Gerlach effect. In addition, the mixture is loaded into a 3D optical lattice to implement the SU(2)×SU(6) Hubbard model. This mixture will open the door to the study of novel quantum phases such as a spinor Bardeen-Cooper-Schrieffer-like fermionic superfluid.

PACS numbers: 03.75.Ss, 67.85.Lm, 37.10.Jk

This is what we need!

Mott insulators of the alkaline earth atoms



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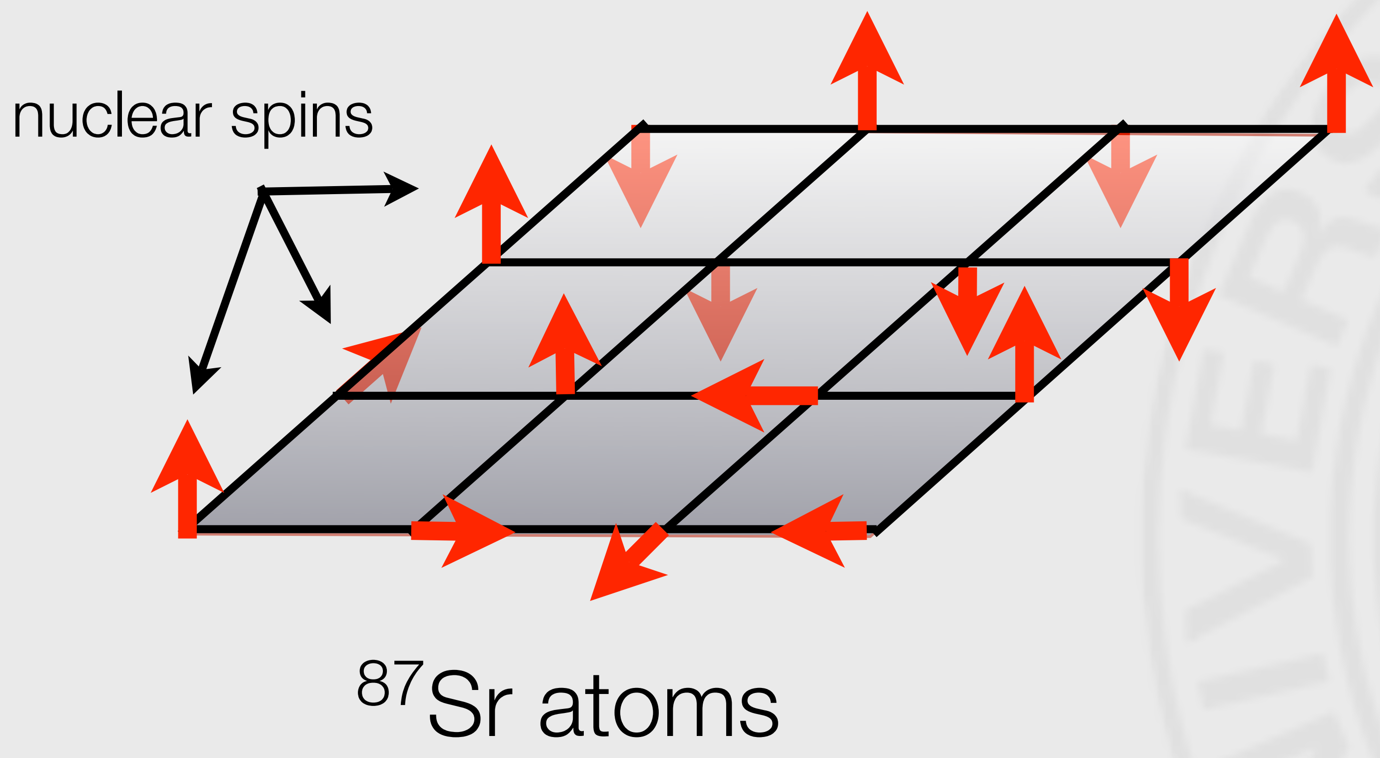

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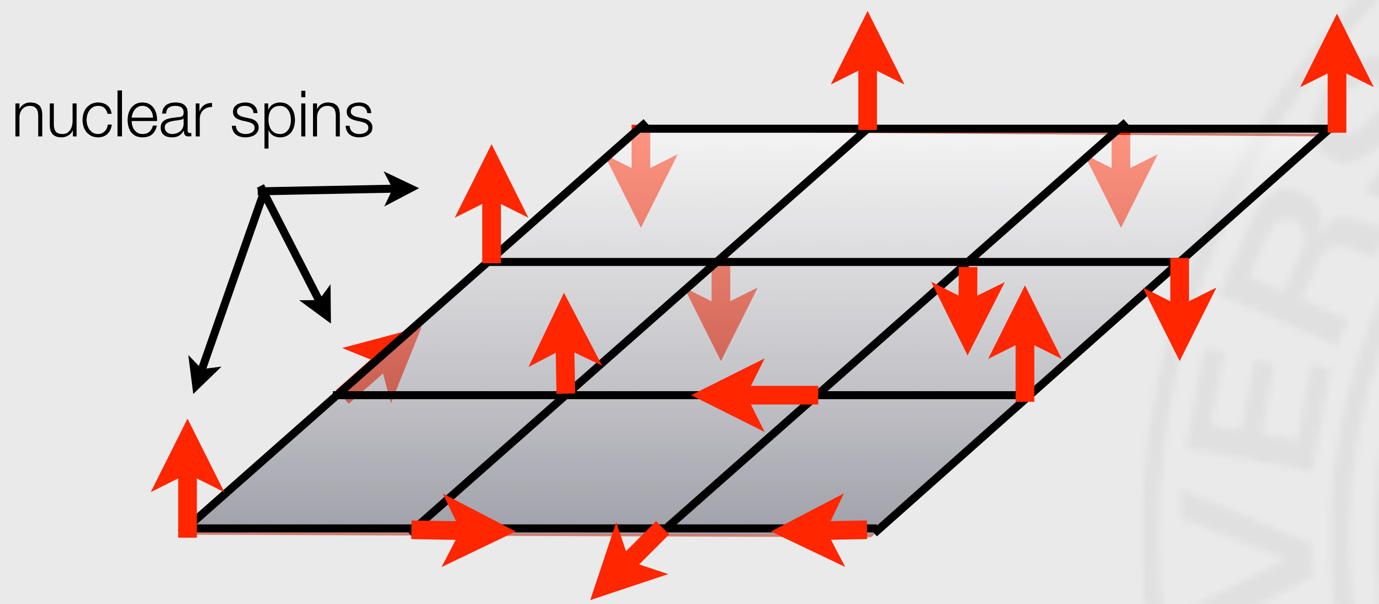

^{87}Sr atoms

$$f^{\dagger}_{i,\alpha}$$

Creates an atom on a site i, with nuclear spin **α**.

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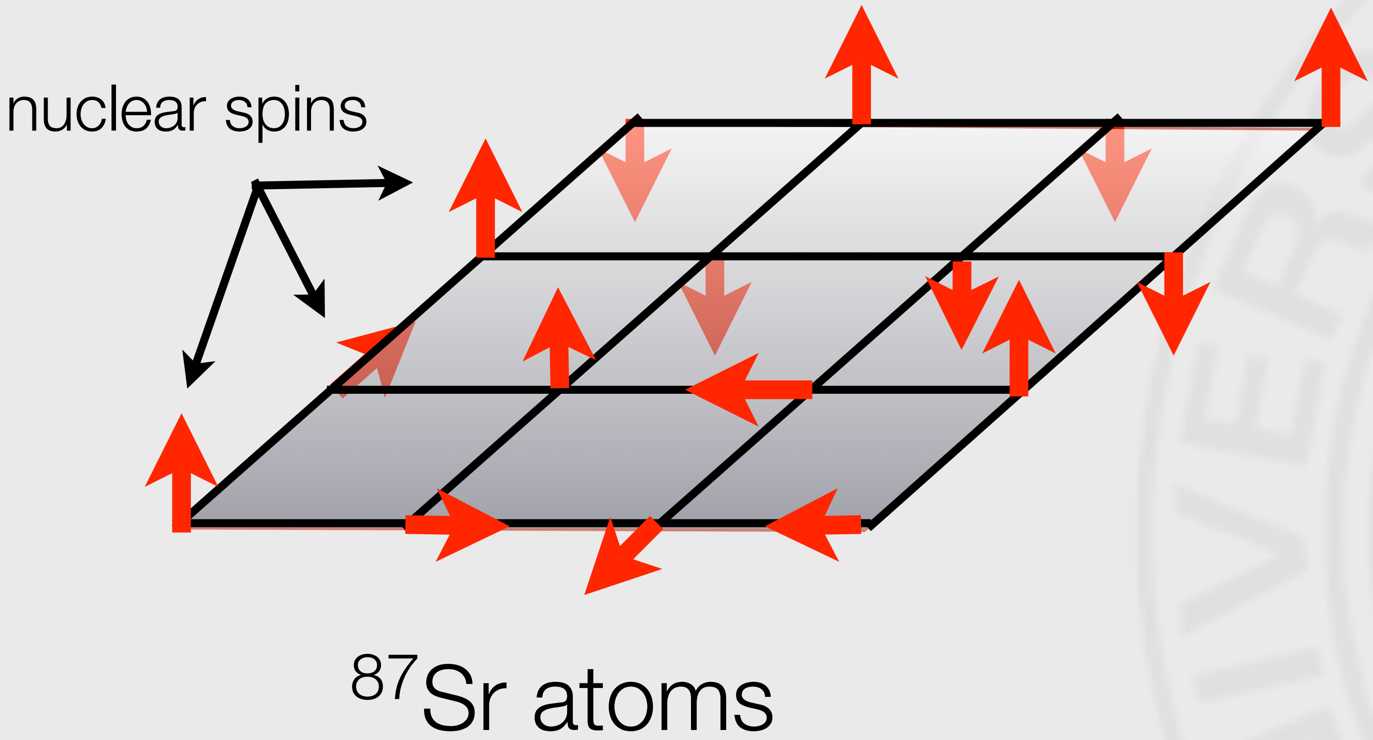

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SU(N) spins

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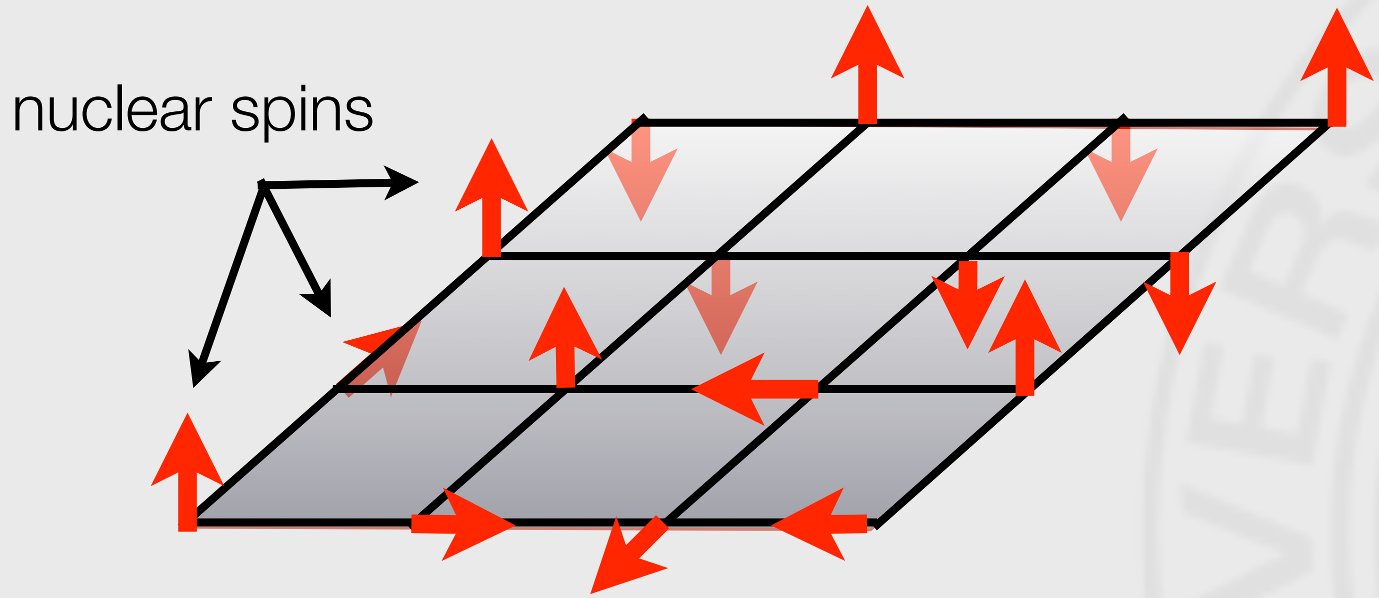

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SU(N) spins

$$\hat{f}_{i,\alpha} \to \sum_{\beta=1}^{N} U_{\alpha,\beta}\hat{f}_{i,\beta}$$

SU(N) symmetry

Bottom line: these are SU(N) spin antiferromagnets

SU(N) antiferromagnets

SU(N) spins are analytically tractable in the large N limit. There is a long history of studying SU(N) spin antiferromagnets, to better understand the usual SU(2) spin antiferromagnets

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S. Sachdev

N. Read

I. Affleck

J. B. Marston

Late 1980's

Careful and numerous studies of the SU(N) antiferromagnets. Papers with many 100's of citations.

SU(N) antiferromagnets

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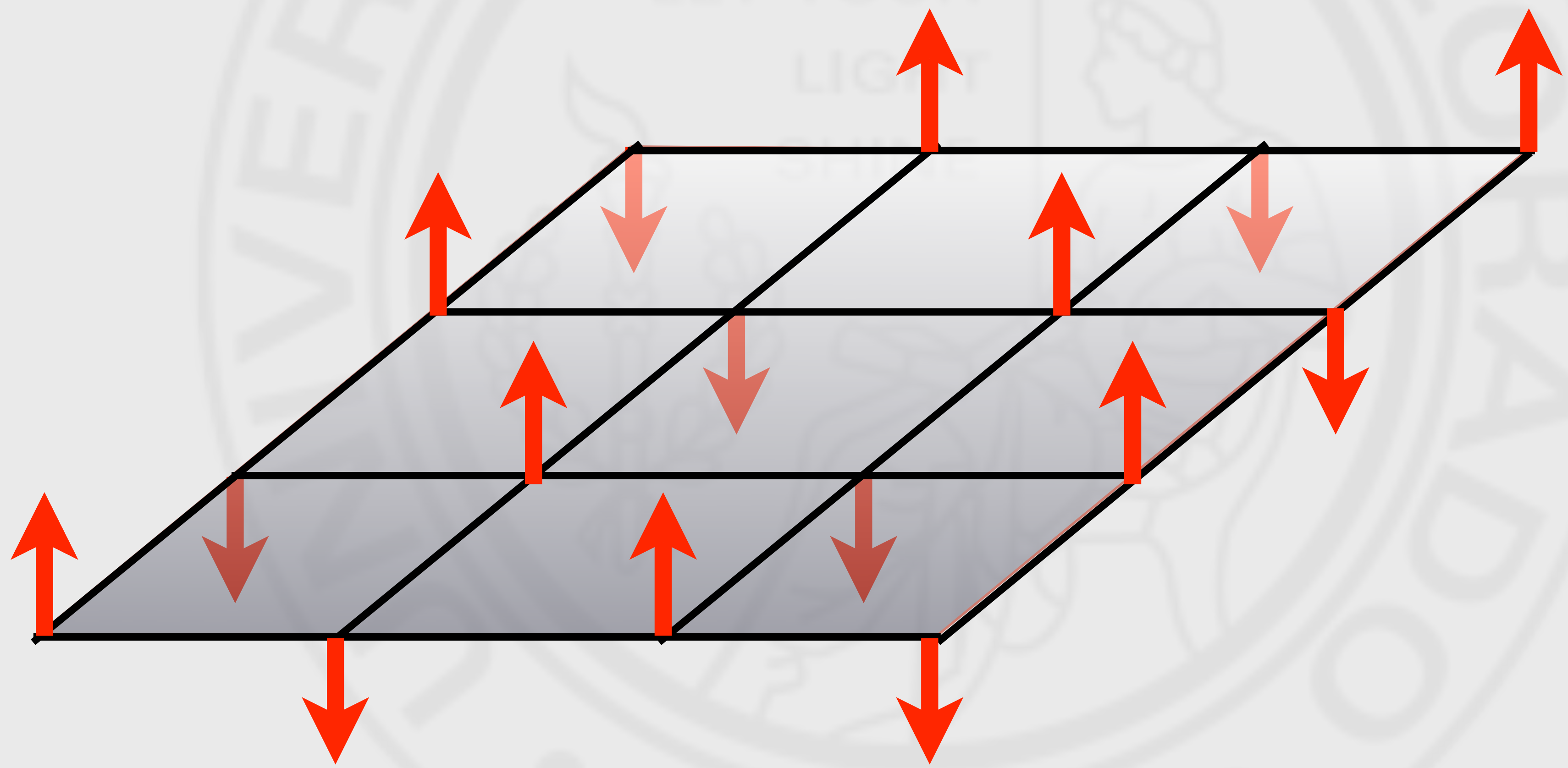
Does this mean we can just look up the answer in these papers and find out all we need to know about SU(N) antiferromagnets and alkaline earth atoms?

NO!

Standard SU(2) antiferromagnet: Néel state

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

SU(2) spins



A collection of spin-1/2s on a square lattice in the presence of the antiferromagnetic interactions at T=0 forms a Néel state with a long range antiferromagnetic order (this is known numerically and experimentally).

The difference between SU(2) and SU(N)

SU(2) spins: two spins-1/2
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$$\alpha, \beta = 1, 2, \ldots, N$$

2 spins:

$$\phi_{\alpha\beta}^{(1,2)} = \phi_{\alpha}^{(1)}\phi_{\beta}^{(2)} - \phi_{\beta}^{(1)}\phi_{\alpha}^{(2)}$$

This is not a singlet, but an antisymmetric *N* by *N* tensor, with *N(N-1)/2* components.

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$\phi_{\alpha\beta\gamma}^{(1,2,3)}$ 3 spins:

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3 spins:

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$$\phi_{\alpha_1\alpha_2\ldots\alpha_N}^{(1,2,\ldots,N)}$$

N spins: finally, scalar!

Prior studies of the SU(N) magnets

All prior studies were designed so that one was able to form singlets from nearby spins

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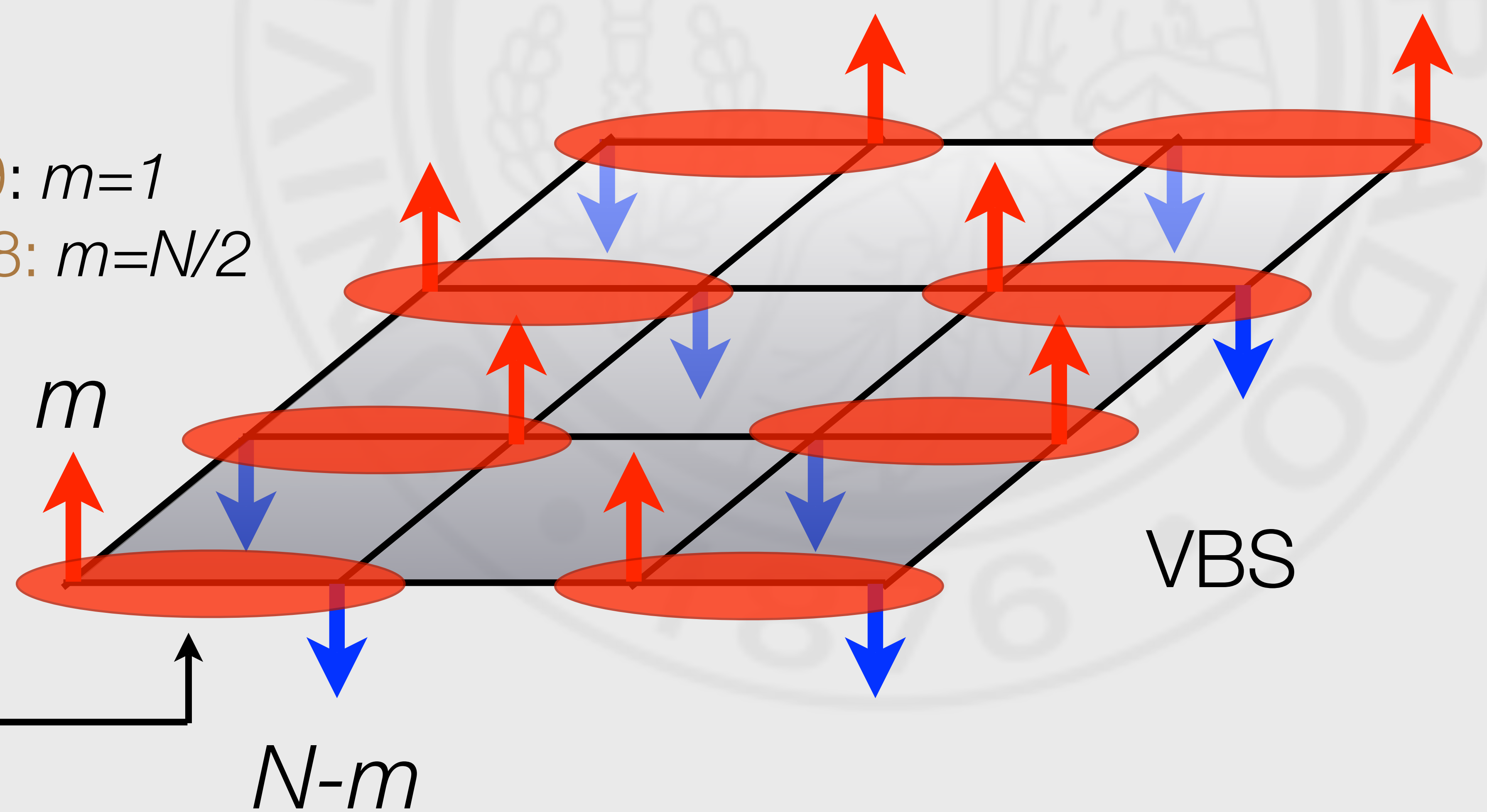

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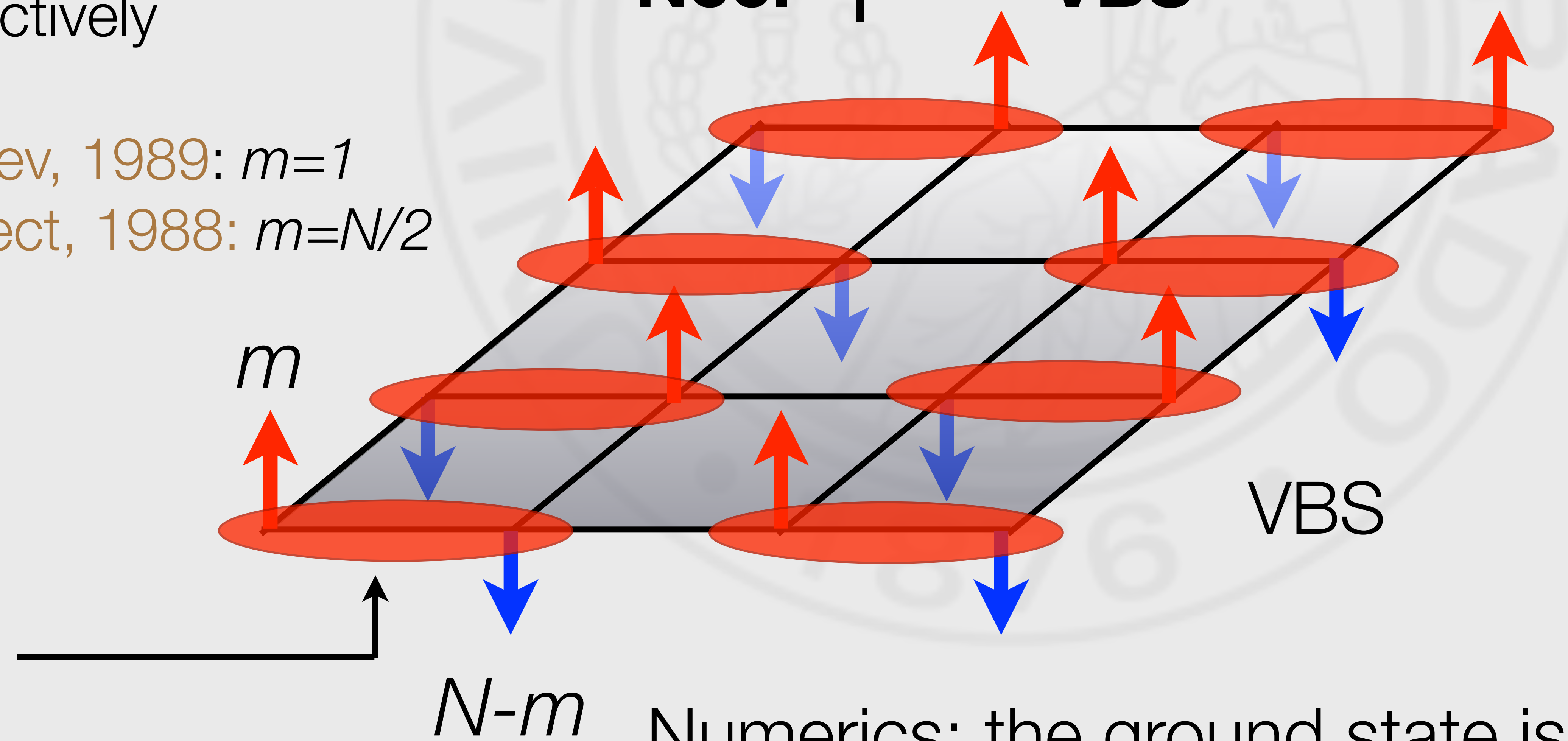


singlets

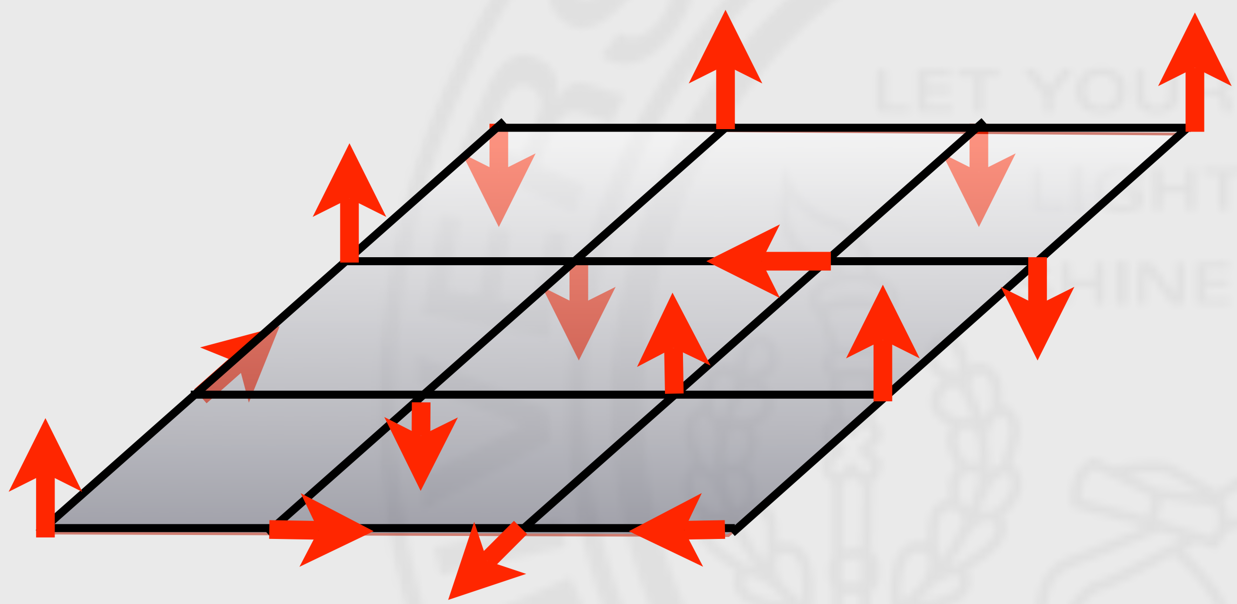
Numerics: the ground state is Néel if $N<4$ VBS if $N>4$

Analytics at large *N*: The ground state is **V**alence **B**ond **S**olid (VBS)

All of this is not relevant for us: we can place one, at most two atoms (SU(N) spins) on each site

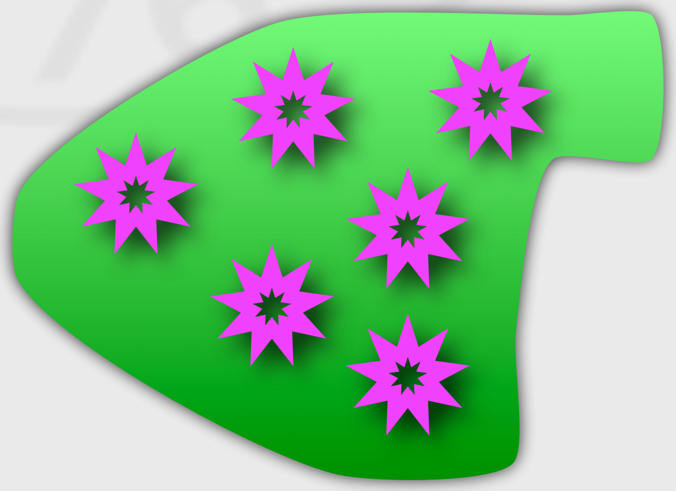
Thus this is a new yet unexplored problem

One atom (or two) per site: experimentally realizable SU(N)



at least *N* (or *N/2*) sites are required to form a singlet

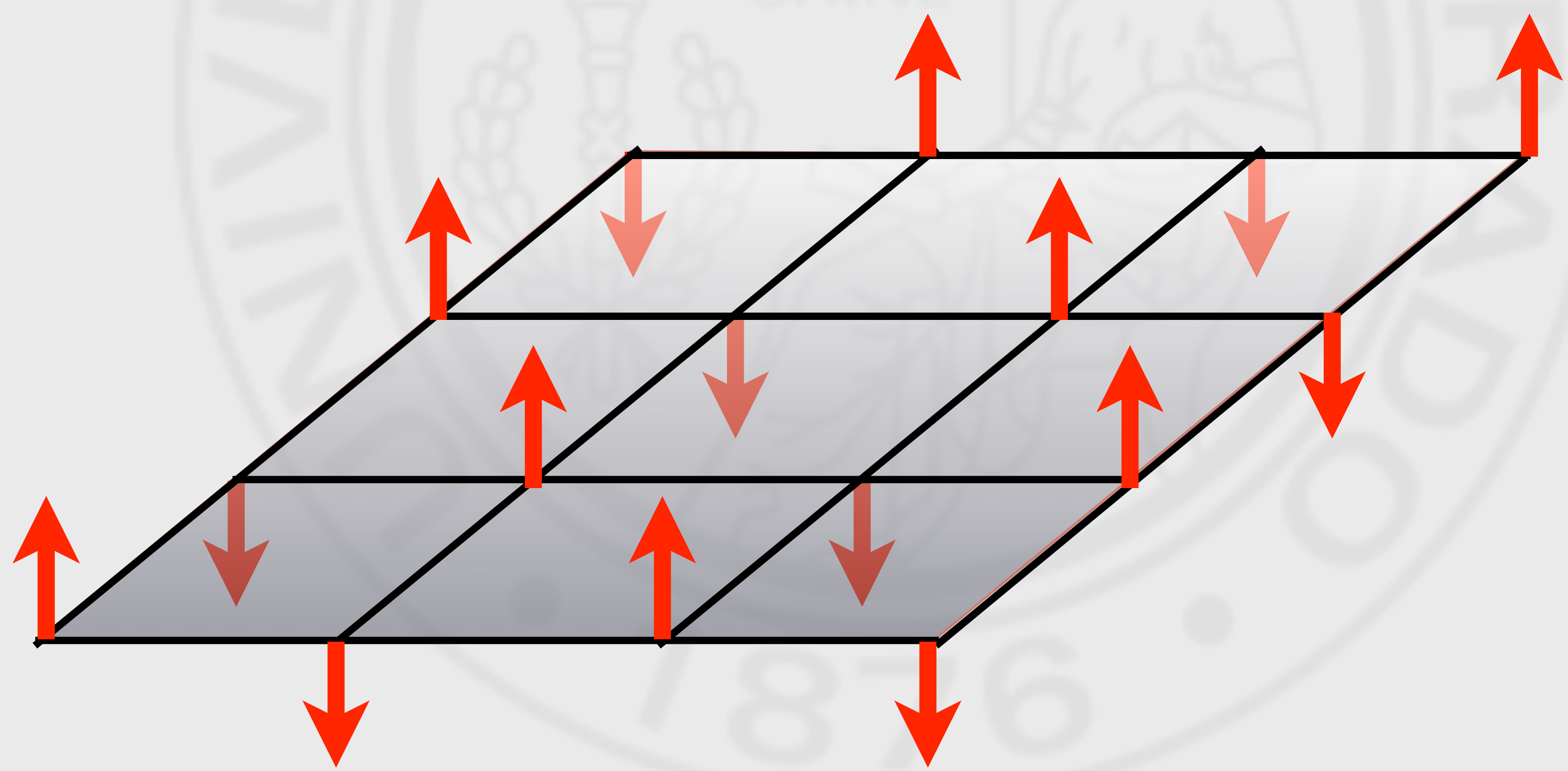
D. Arovas (2008) calls these N-simplexes.



Quasiclassical analysis

Quasiclassical version: SU(2)

We'd like to imagine spins as arrows $\vec{S}$



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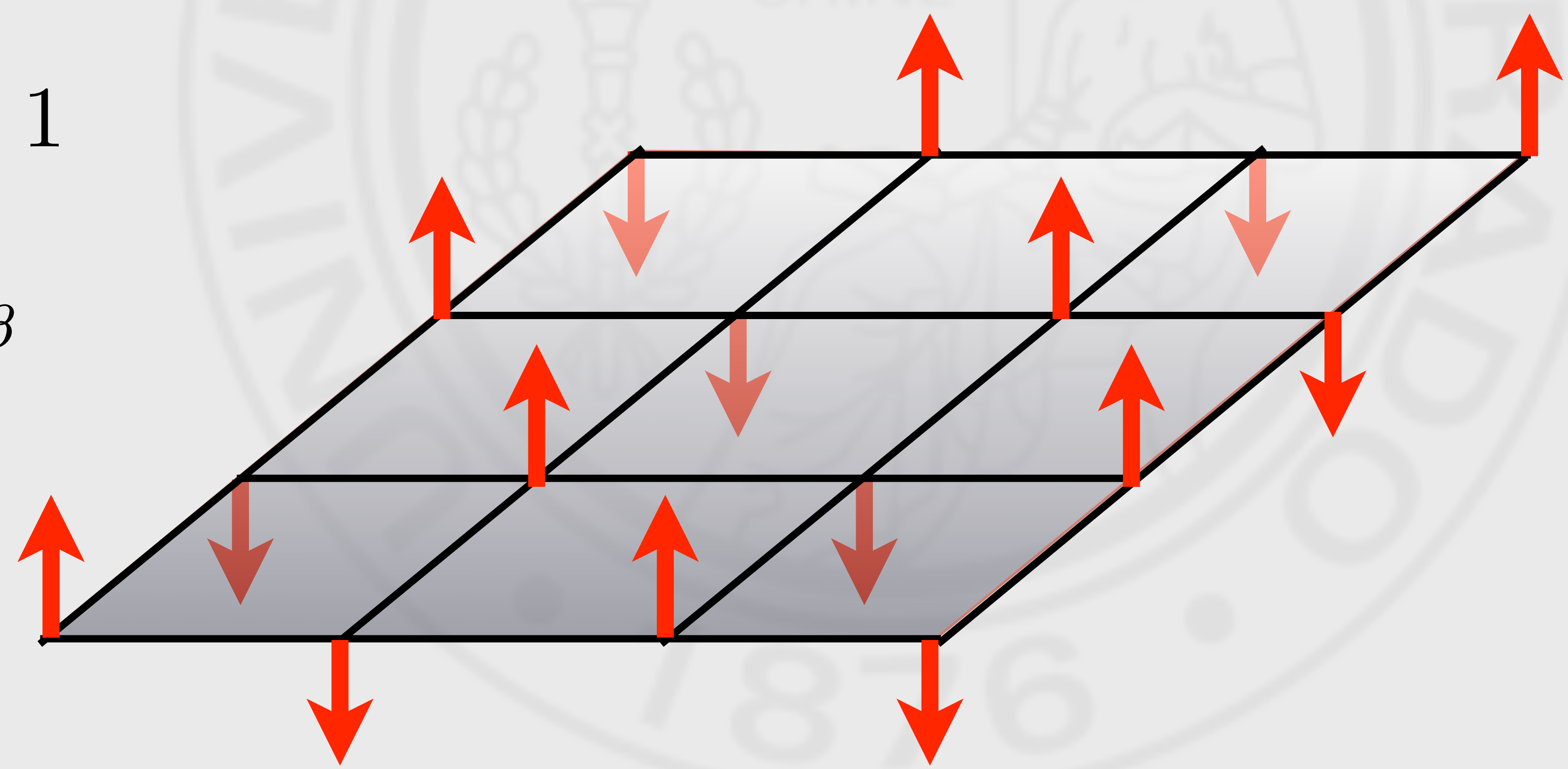

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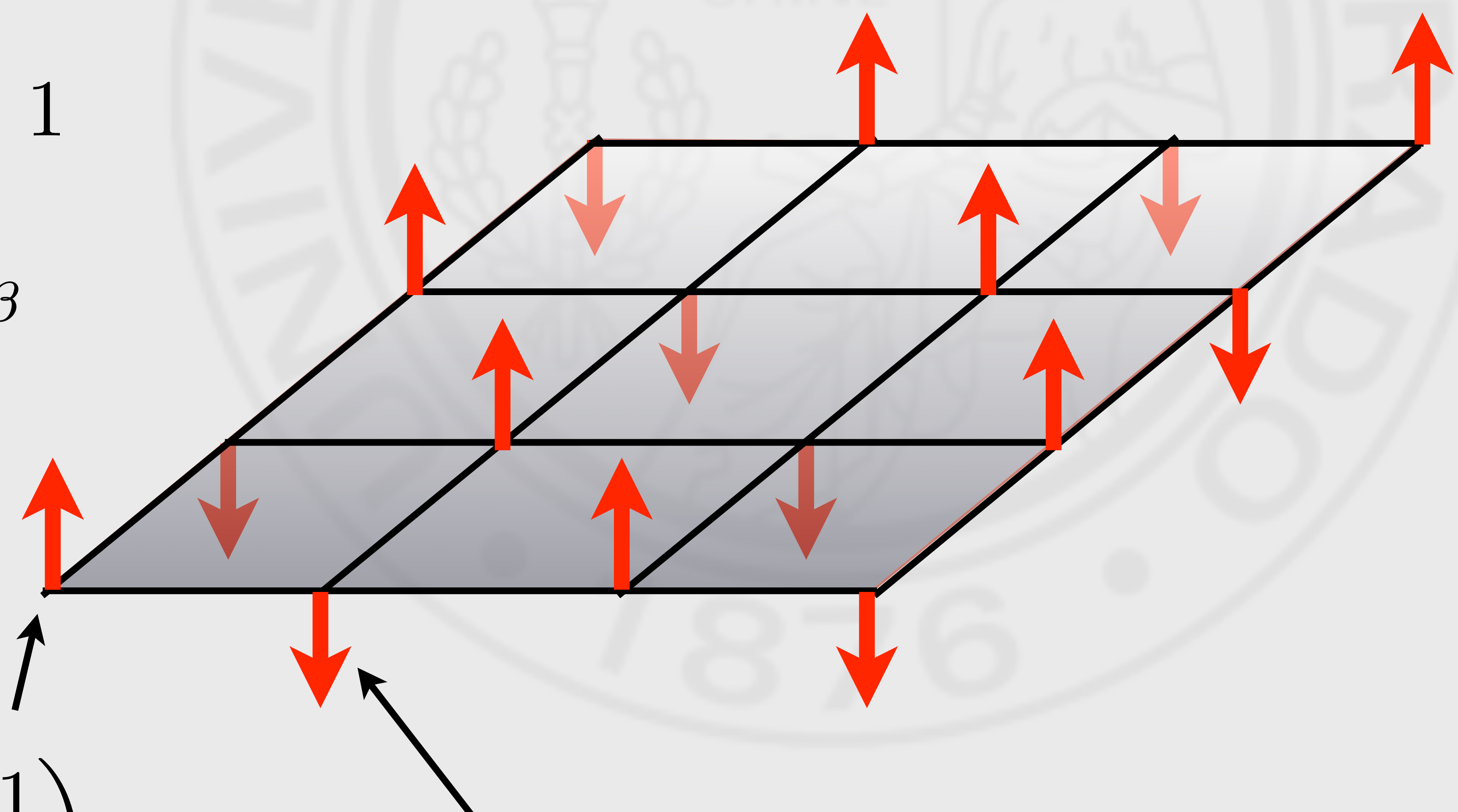
$$H \sim J \sum_{\langle ij \rangle} |z^*_i \cdot z_j|^2$$



$$z = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$z = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Antiferromagnetism

SU(N) quasiclassical spins

SU(N): each spin is a complex vector

$$z_\alpha, \ \ \alpha = 1, 2, \ldots, N$$

$$H \sim J \sum_{\langle ij \rangle} \left| z_i^* \cdot z_j \right|^2$$

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"Orthomagnetism"
(spins are trying to all be orthogonal)

Frustration: large number of classical ground states

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Counting classical ground states (Moessner & Chalker):

z : 2*N*-2 real degrees of freedom $\quad D = 2N_s(N-1)$

$z_i^* \cdot z_j = 0$ 2 constraints per bond $\quad C \leq 4N_s$

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z : *2N-2* real degrees of freedom $\quad D = 2N_s(N-1)$

$z_i^* \cdot z_j = 0$ 2 constraints per bond $\quad C \leq 4N_s$

ground state degeneracy $\quad G = D - C \geq 2N_s(N-3)$

extensive classical ground state if $\quad \underline{N > 3}$

SU(N) quasiclassical spins

This implies that the standard methods to treat quantum antiferromagnets (sigma models) do not apply.
One also expects that magnetic order is unlikely.

Large N methods

Large N methods

Taking *N* to infinity is problematic: the number of spins required to form a singlet goes to infinity too.

Proposal: Let us put $m = \frac{N}{k}$ atoms on each site.

The number of sites required to form a singlet is now *k* and is *N* independent. Then take *N* to infinity.

Carrying out this procedure results in the chiral spin liquid ground state if *k*>*4*.

M. Hermele, VG, A.-M. Rey (2009)

Saddle point approximation

$$H = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha, \beta = 1, \ldots, N} \hat{f}^\dagger_{j,\beta} \hat{f}_{i,\beta} \hat{f}^\dagger_{i,\alpha} \hat{f}_{j,\alpha}$$

convenient for large N limit

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convenient for large N limit

Habbard-Stratonovich hopping

$$S = \int d\tau \left[\sum_i \left\{ \sum_\alpha \bar{f}_{i,\alpha} \partial_\tau f_{i\alpha} + i\lambda_i \left(\sum_\alpha \bar{f}_{i,\alpha} f_{i,\alpha} - \frac{N}{k} \right) \right\} + \sum_{<ij>} \left\{ \chi_{ij} \sum_\alpha \bar{f}_{i\alpha} f_{j\alpha} + \text{H.c.} + N \frac{|\chi_{ij}|^2}{J} \right\} \right]$$

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Saddle point equations (exact in the large *N* limit)

$$\frac{1}{k} = \left\langle \hat{f}^\dagger_{i,\alpha} \hat{f}_{i,\alpha} \right\rangle \qquad \chi_{ij} = -J \left\langle \hat{f}^\dagger_{j,\alpha} \hat{f}_{i,\alpha} \right\rangle$$

Fermions try to arrange their hopping dynamically to minimize their energy

Saddle point approximation

$$H = -\frac{J}{N} \sum_{\langle ij \rangle, \alpha, \beta = 1, \dots, N} \hat{f}^\dagger_{j,\beta} \hat{f}_{i,\beta} \hat{f}^\dagger_{i,\alpha} \hat{f}_{j,\alpha}$$

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$$\chi_{ij} = \chi^{(0)}_{ij} e^{iA_{ij}}$$

saddle point

fluctuations

$$S_{\mathrm{eff}}[A_{ij}]$$

Fermions try to arrange their hopping dynamically to minimize their energy

Saddle points for k=2, 3, 4

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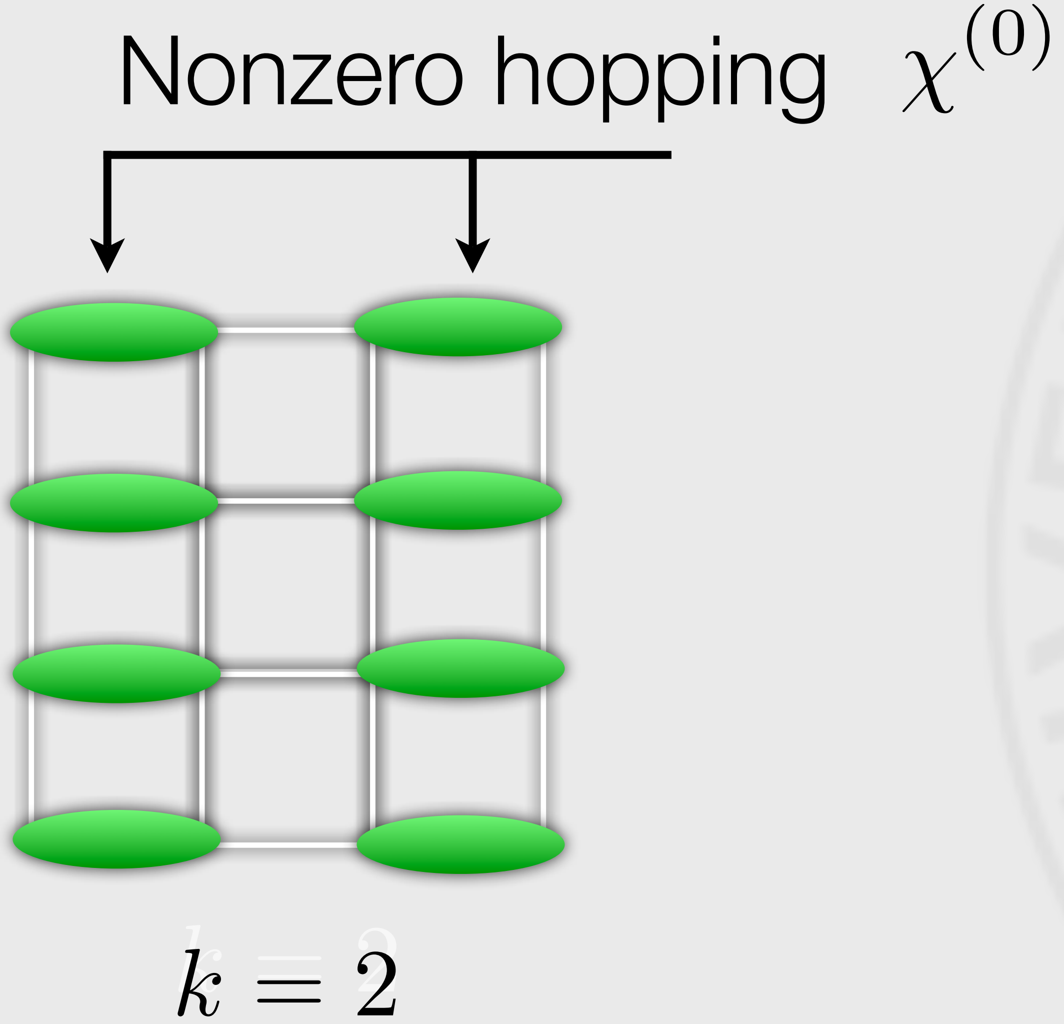
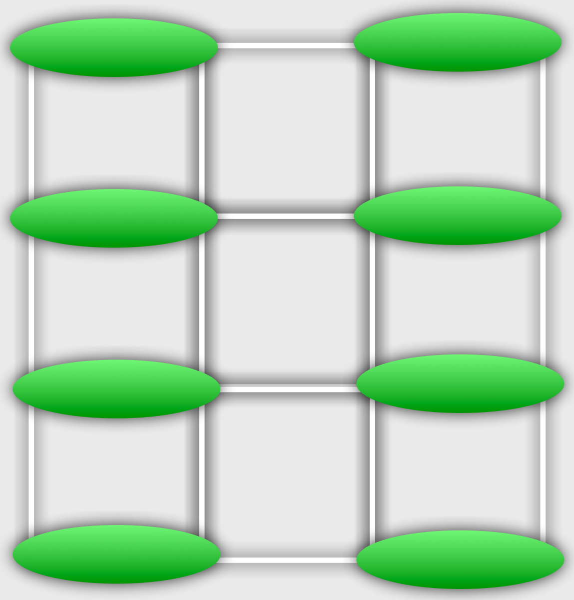


Spins pair up to form 2-spin singlets
This state (VBS) was found by Affleck & Marston in 1989

Saddle points for k=2, 3, 4

Nonzero hopping $\chi^{(0)}$



$k = 2$

Spins pair up to form 2-spin singlets
This state (VBS) was found by Affleck & Marston in 1989

$k = 3$

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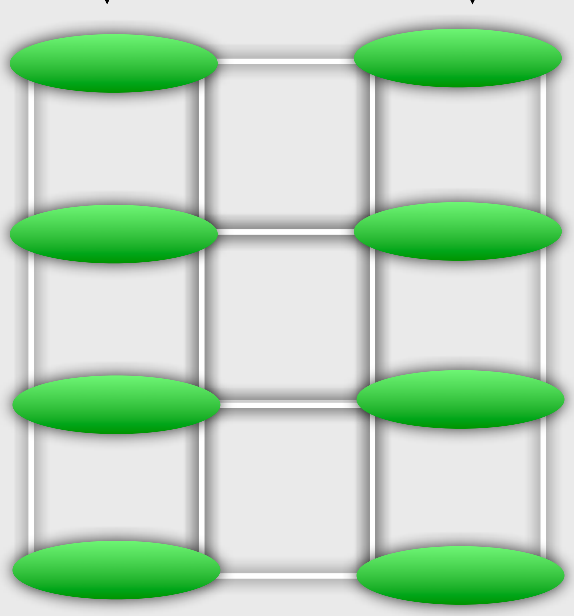
Spins form 6-spin singlets
(new result)

Saddle points for k=2, 3, 4

Nonzero hopping $\chi^{(0)}$



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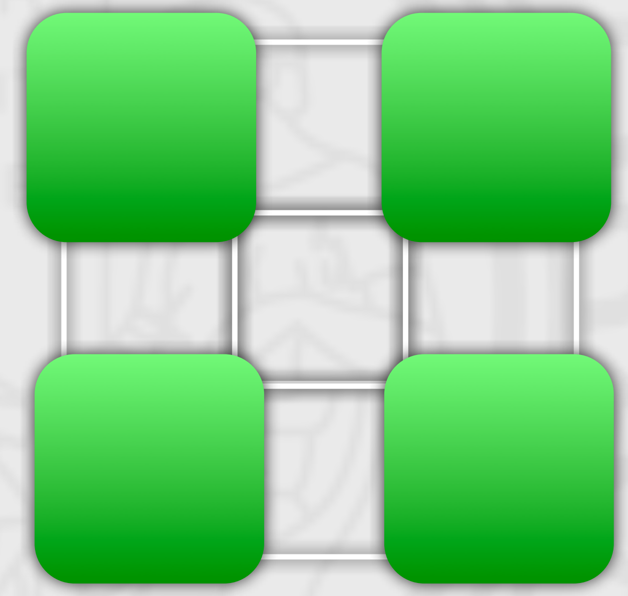


$k = 4$

Spins form 4-spin singlets
(new result)

Saddle points for *k>4*

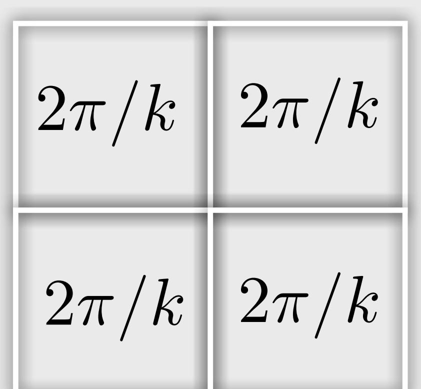
To minimize their energy, fermions attempt to organize hoppings so that they completely fill a band

The filling fraction for fermions with one of *N* spin components is

$$\nu = \frac{1}{N}\frac{N}{k} = \frac{1}{k}$$

Fermions would like to form a closed band with N_s/k states

Our result: the best way to do that is by arranging a "magnetic flux" of $2\pi/k$ per plaquette and fill the lowest Landau level.



M. Hermele, VG, A.-M. Rey (2009)

Chiral Spin Liquid

Chiral Spin Liquid (CSL)

Wen, Wilczek & Zee Kalmeyer & Laughlin

Proposed in 1989

A state of magnets
without any magnetic order (spin liquid),
but breaking parity and time reversal invariance (chiral).

Has to be described by a Chern-Simons theory.

(Local) Hamiltonians whose ground state would be CSL
were unknown until now

Chiral spin liquid (k>4)

$$H = \sum_{<ij>,\alpha} \chi_{ij}^{(0)} \left(\hat{f}_{i,\alpha}^{\dagger} \hat{f}_{j,\alpha} + \text{H.c.} \right)$$

magnetic field with 1/k flux through plaquette

like quantum Hall effect

$$\chi_{ij} = \chi_{ij}^{(0)} e^{iA_{ij}}$$

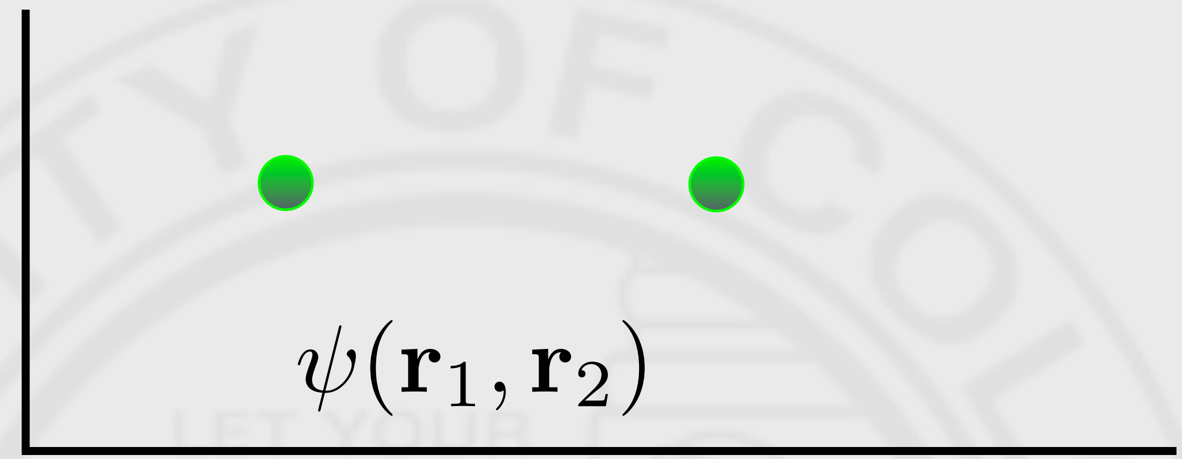
$$S_{\text{eff}} = \frac{N}{4\pi} \int d^2x dt\, \epsilon_{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

Level *N* Chern-Simons theory

Fermions acquire fractional statistics with the angle θ:

$$\theta = \pi + \frac{\pi}{N} \qquad N = k, 2k, 3k, \ldots$$

Chiral spin liquid (k>4)



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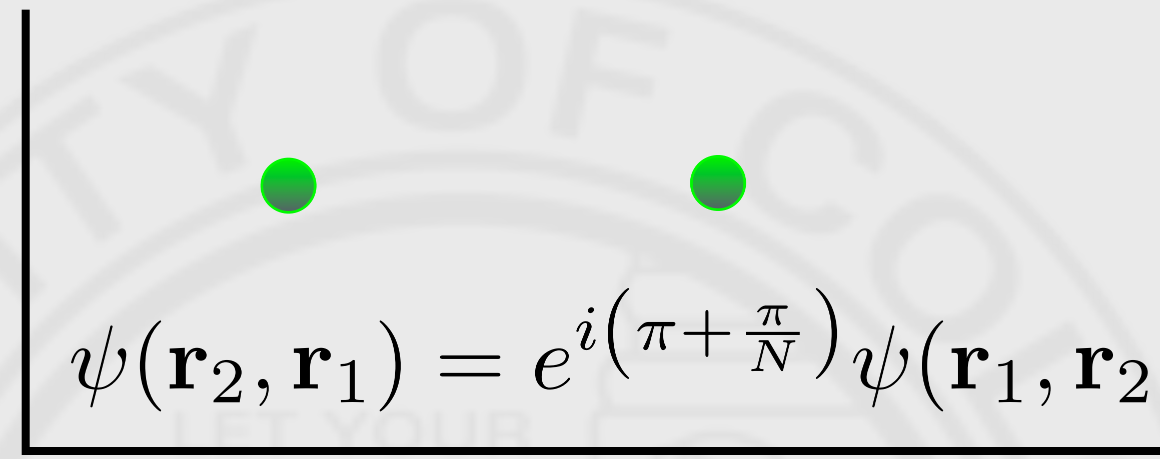

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Level *N* Chern-Simons theory

Fermions acquire fractional statistics with the angle **θ**:

$$\theta = \pi + \frac{\pi}{N} \qquad N = k, 2k, 3k, \ldots$$

Who's the carrier of the statistics?

$$\hat{S}^{\alpha}_{\beta}(i) = \hat{f}^{\dagger}_{i,\alpha}\hat{f}_{i,\beta}$$

The spin itself is not fractional

The fermions are fractional, but with each site containing exactly one atom, the fermionic atoms don't have any dynamics, fractional or otherwise

Let's create "holes" - empty atomless sites on the lattice

$$\hat{f}_{i,\alpha} = \hat{b}^{\dagger}_{i}\hat{c}_{i,\alpha}$$

"spinon" - fermion carrying spin

"holon" - boson carrying atom number

atom

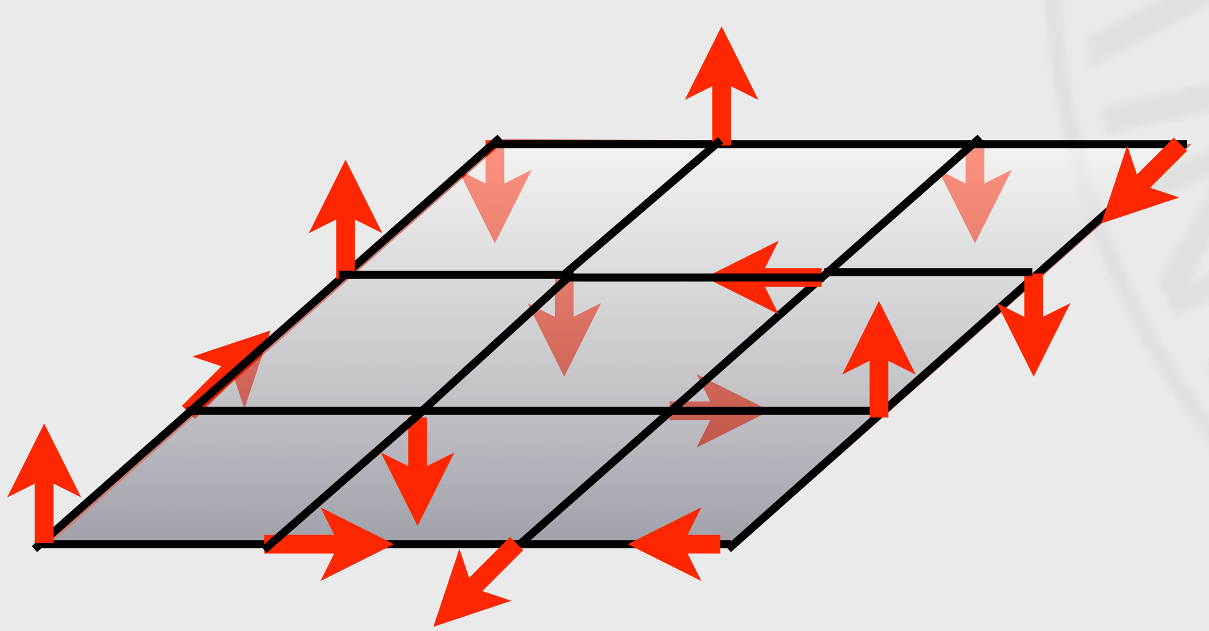
Well known construction from high T_c theories - only here justified by large *N*.

Both spinons and holons are fractional, but only holons respond to an external potential

P.A. Lee, N. Nagaosa (1992)

Scenario to create fractional excitations

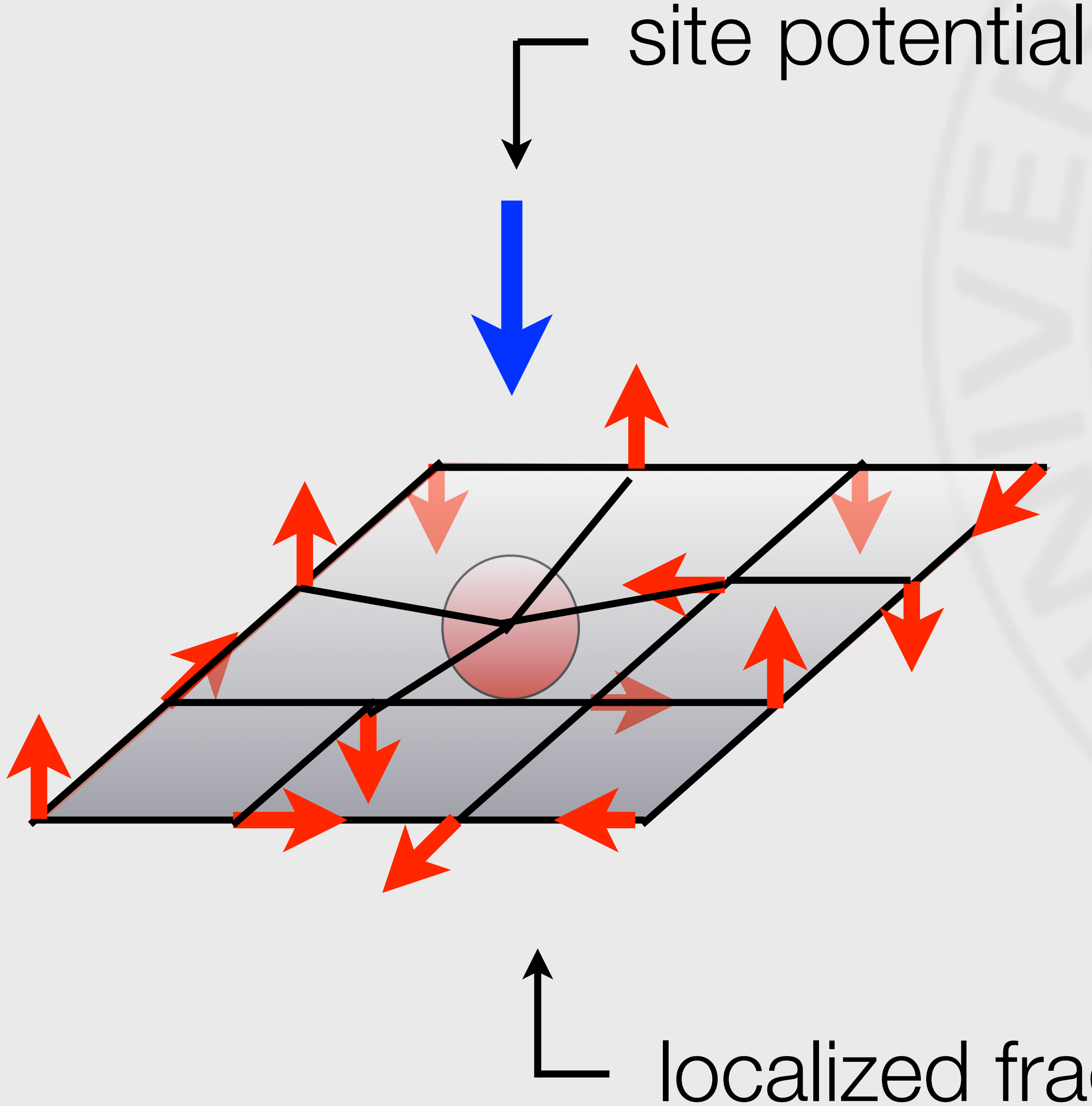
site potential



Lowering the potential at one site localizes a fractional particle at that site.

localized fractional holon

Scenario to create fractional excitations



Lowering the potential at one site localizes a fractional particle at that site.

Non-Abelian chiral spin liquid

Place two species of fermionic atoms on each site, in two distinct electronic states $^1S_0$ and $^3P_0$.

They form either antisymmetric (boring) or symmetric (interesting) state depending on the relative strength of interaction constants (10 author paper)

$a, b = {}^1S_0, {}^3P_0$ labels species

$$|s\rangle = \frac{1}{\sqrt{2}}\left(\hat{f}^\dagger_{a,\alpha}\hat{f}^\dagger_{b,\beta} + \hat{f}^\dagger_{b,\alpha}\hat{f}^\dagger_{a,\beta}\right)|0\rangle$$ on every site of the lattice

$$H = \sum_{\langle ij\rangle,a,b,\alpha,\beta} \hat{f}^\dagger_{i,a,\alpha}\hat{f}_{i,a,\beta}\hat{f}^\dagger_{j,b,\beta}\hat{f}_{j,b,\alpha}$$

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a, *b* = $^1S_0$, $^3P_0$ labels species

$${\chi_{ij}^{(0)}}^{ab} = \delta_{ab} \chi_{ij}^{(0)} \longleftarrow \text{magnetic field}$$

$$S_{CS} = \frac{N}{4\pi} \mathrm{Tr} \int d^2x dt \, \epsilon_{\mu\nu\rho} \left[A_\mu \partial_\nu A_\rho + \frac{2}{3} A_\mu A_\nu A_\rho \right]$$

This is non-Abelian Chern-Simons $SU(2)_N$ theory!

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Topological quantum computing with $SU(2)_{10}$!

Non-Abelian chiral spin liquid

Topological quantum computing

A. Kitaev, 1997

Non-Abelian particles
are perfect qubits

Topological quantum computing

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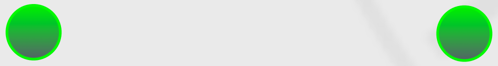
Non-Abelian particles are perfect qubits



$$\psi_\alpha(\mathbf{r}_1, \mathbf{r}_2)$$

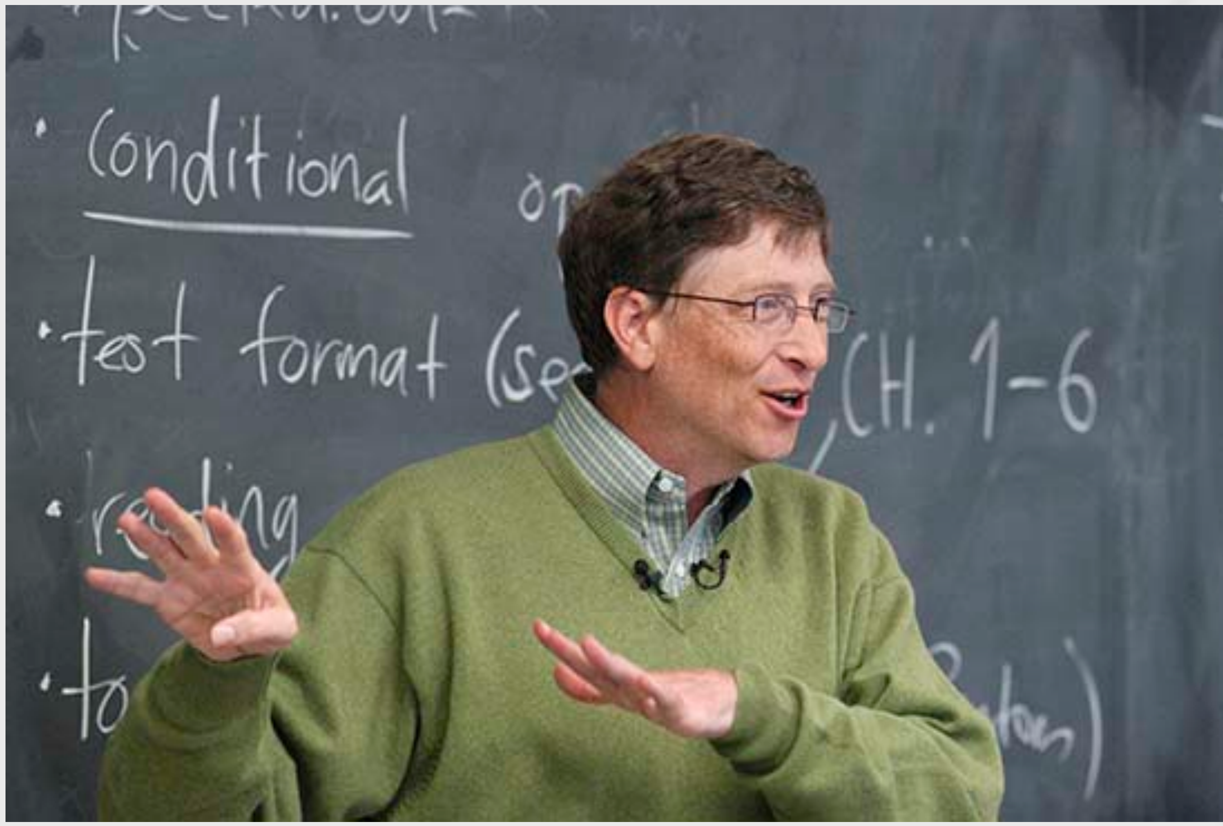
Topological quantum computing

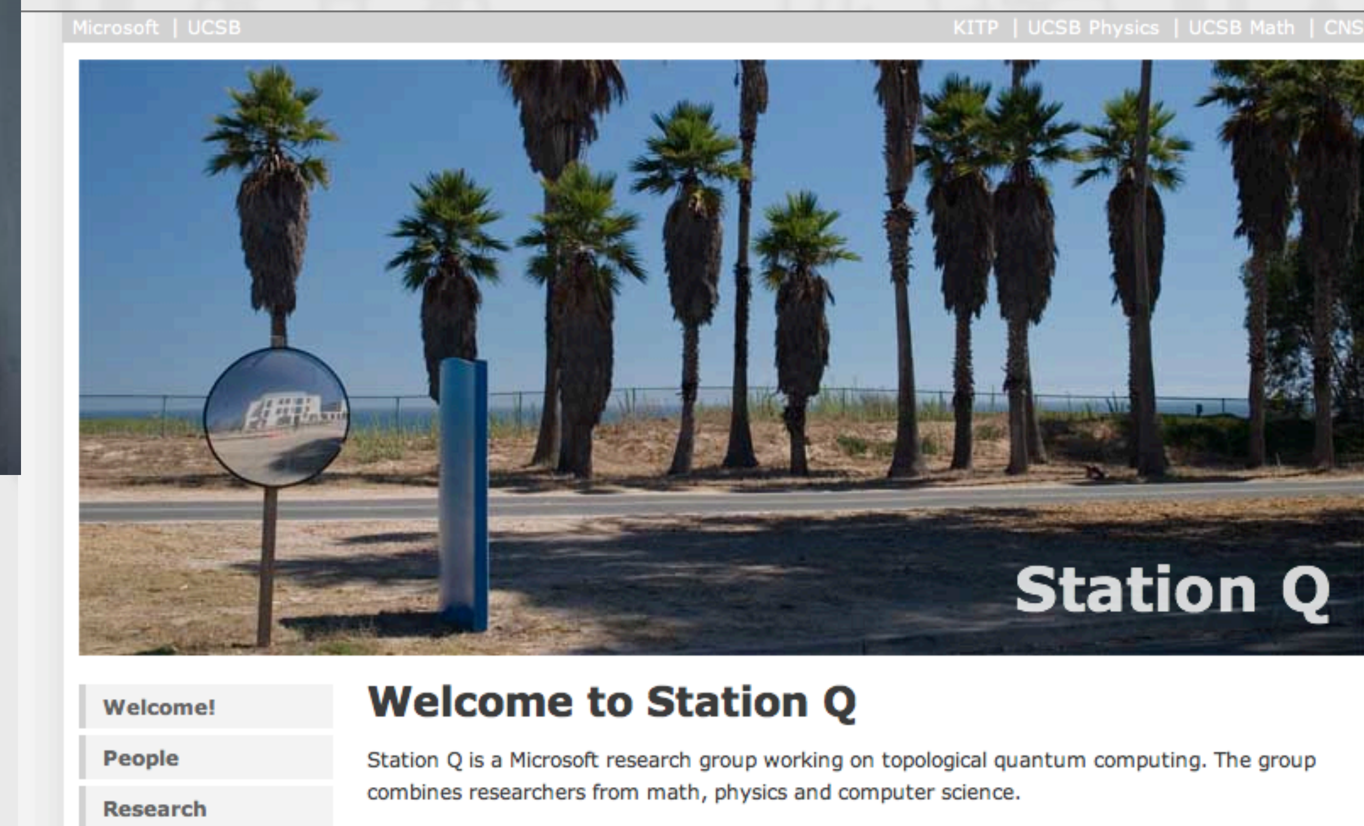
A. Kitaev, 1997

Non-Abelian particles
are perfect qubits

$$\psi_\alpha(\mathbf{r}_2, \mathbf{r}_1) = \sum_\beta U_{\alpha,\beta}\psi_\beta(\mathbf{r}_1, \mathbf{r}_2)$$

Topological quantum computing

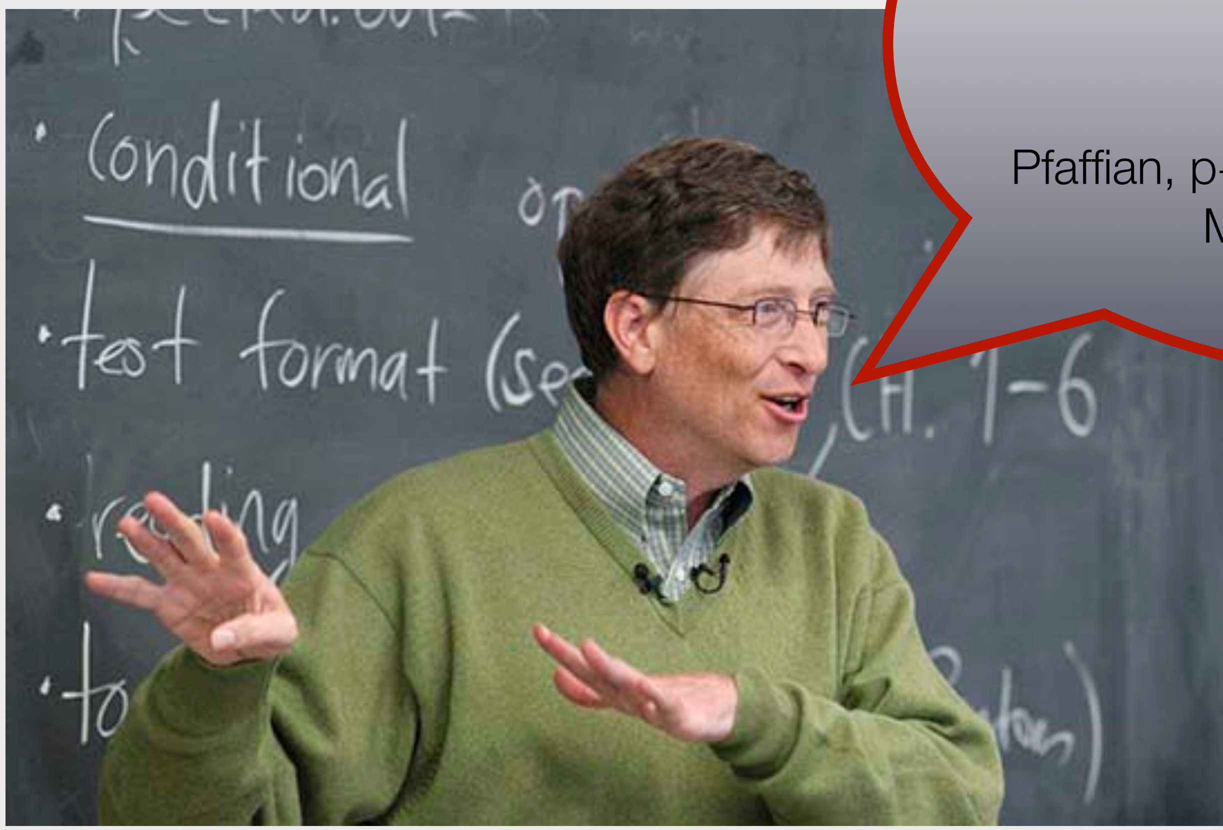


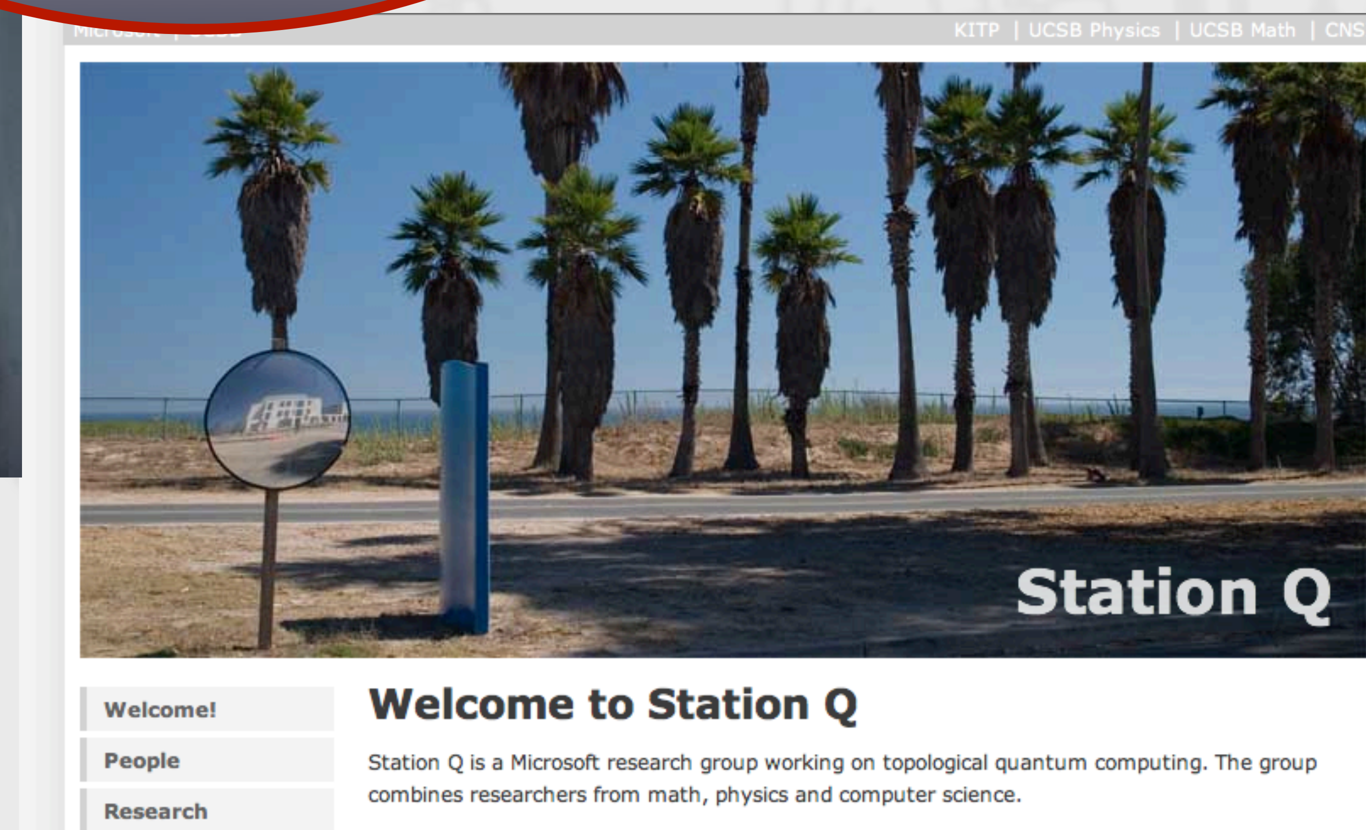


Topological quantum computing

We wish we had an $SU(2)_2$

Pfaffian, p+ip superconductor, all those Majorana fermions...





How about $SU(2)_{10}$!

Topological insulator

a competing state

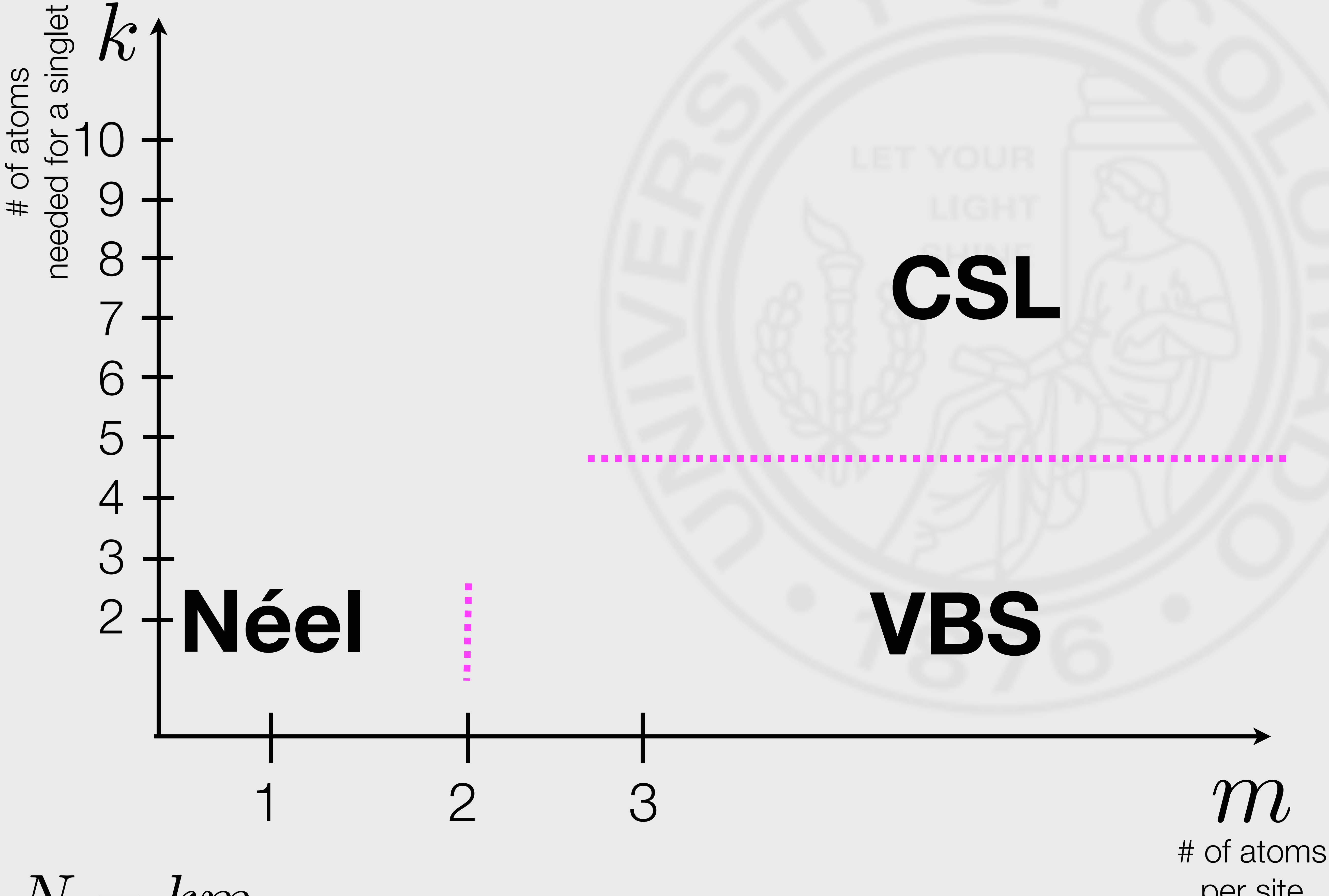
$$H = \sum_{ij,a,b,\alpha} {\chi_{ij}^{(0)}}^{ab} \hat{f}^{\dagger}_{i,a,\alpha} \hat{f}_{j,b,\alpha}$$

$${\chi_{ij}^{(0)}}^{ab} = (-1)^a \delta_{ab} \chi_{ij}^{(0)}$$

opposite magnetic field for opposite spin polarizations

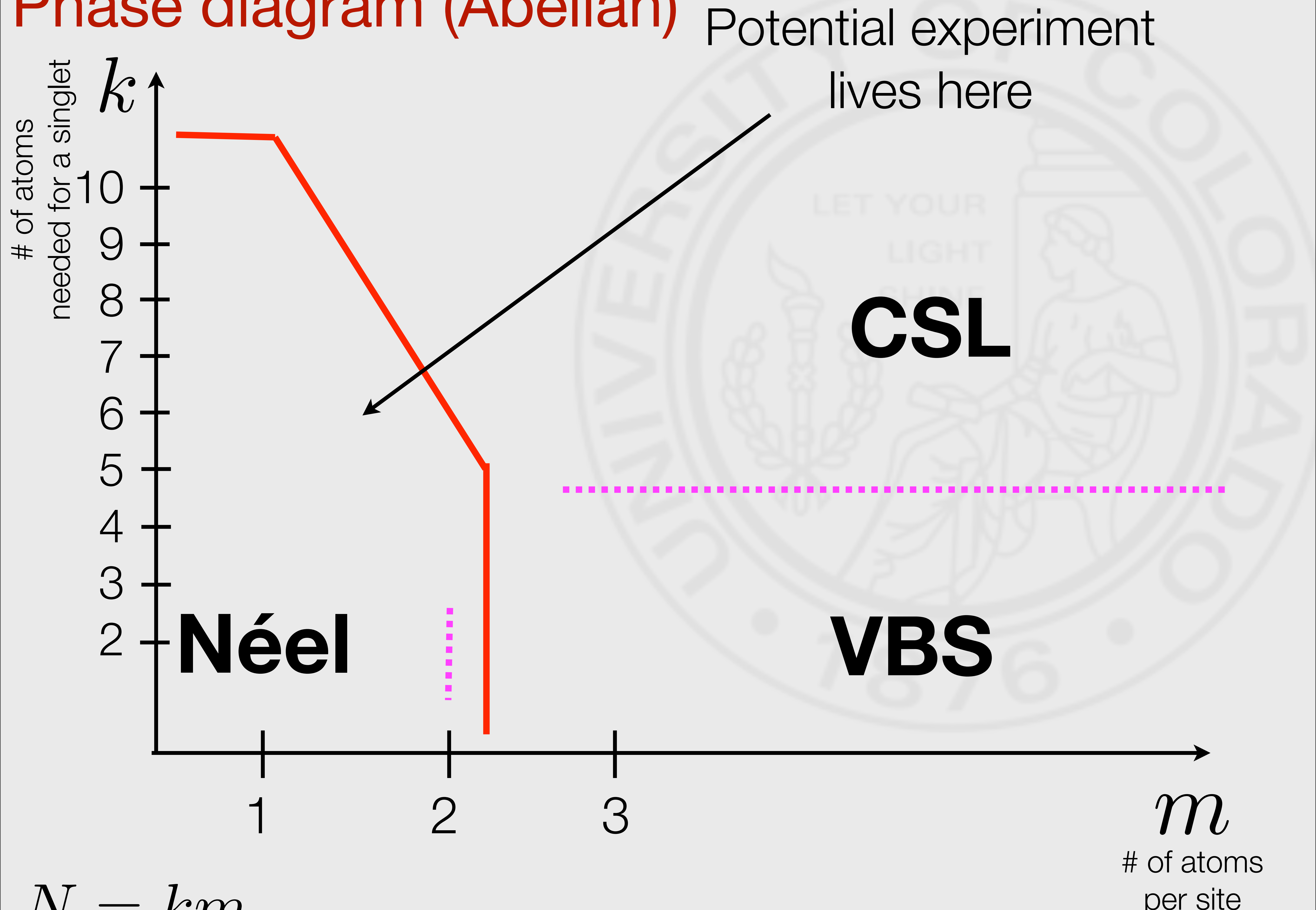
$$S_{\text{eff}} = \frac{N}{4\pi} \sum_a (-1)^a \int d^2x dt \, \epsilon_{\mu\nu\rho} A^a_\mu \partial_\nu A^a_\rho$$

Phase diagram (Abelian)



$N = km$

Phase diagram (Abelian)



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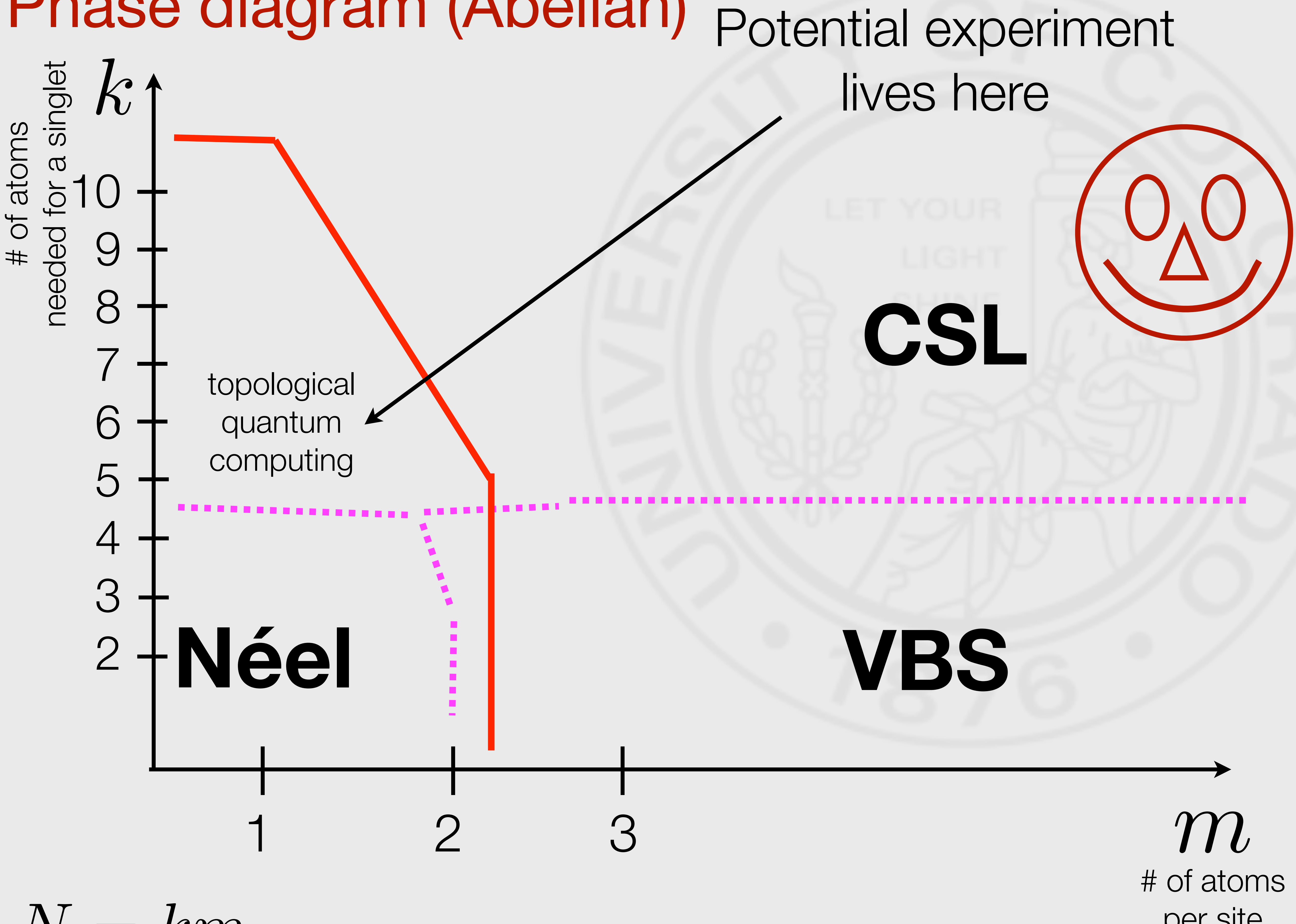
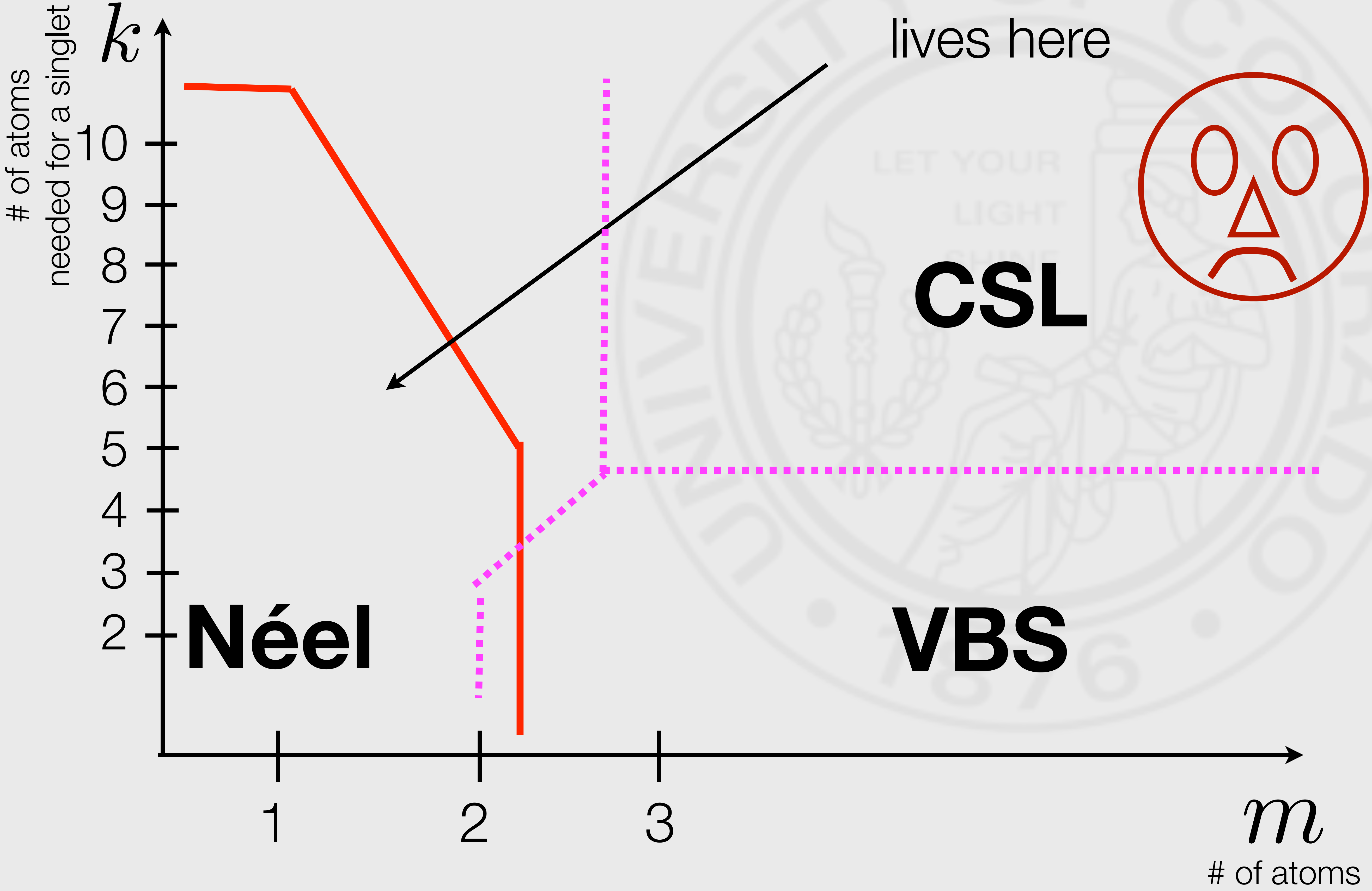


$$N = km$$

Phase diagram (Abelian)

Potential experiment lives here



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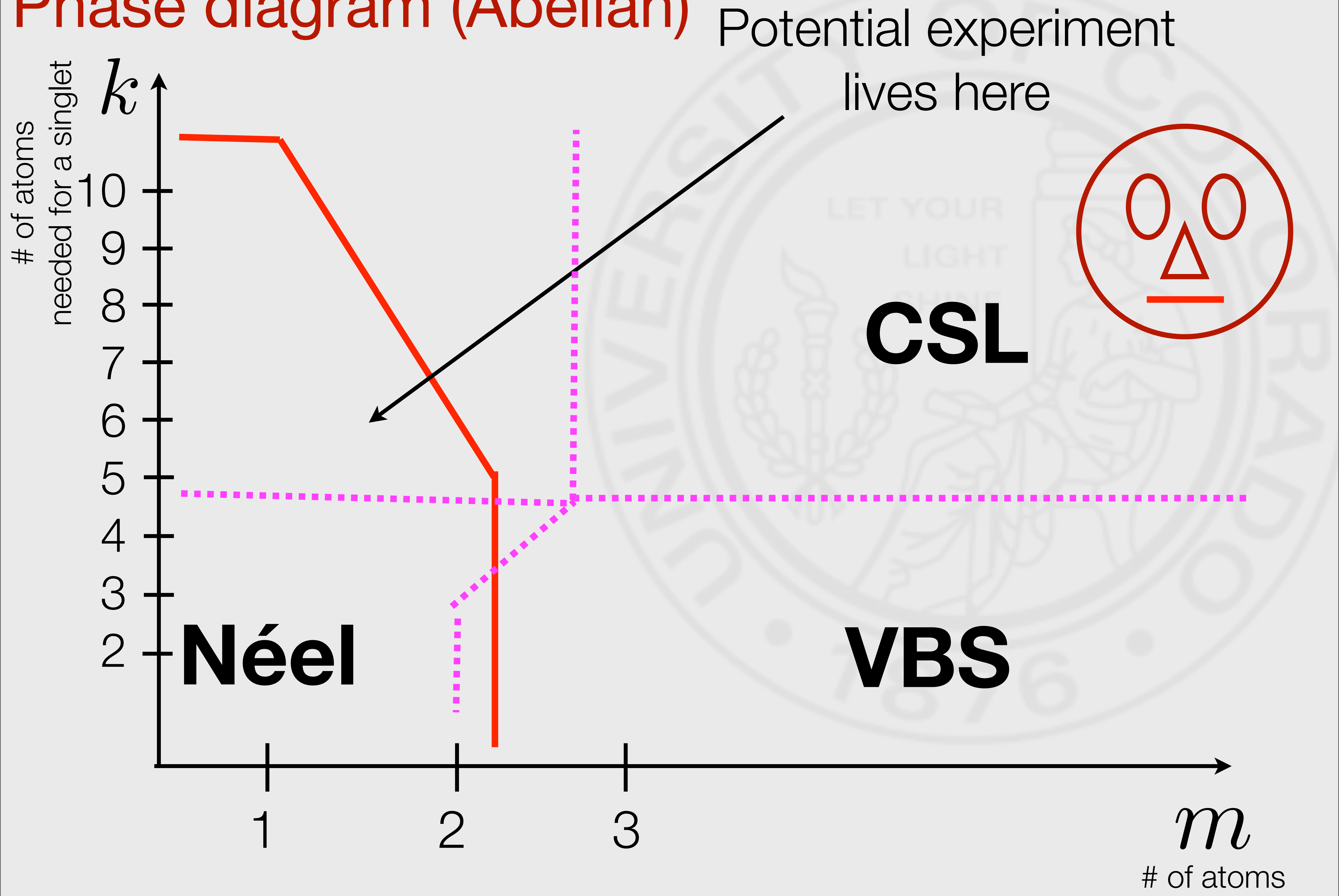

$N = km$

Status of numerics

- Numerical methods are the only way to access the experimentally relevant $m=1$ column
- QMC - sign problem for $k>2$, at least in 2D
- Other 2D numerical methods?

Conclusions

- SU(N) magnets are a useful theoretical construct due to the existence of the large *N* techniques

- SU(N) magnets can have phases going beyond the phases of the SU(2) magnets

- Nuclear spin of the alkaline earth atoms a perfect realization of the SU(N) spin - so far lacking in condensed matter

- A version of the SU(N) magnets particularly well suited to realization by the alkaline earth atoms forms chiral spin liquids, a state of matter with fractionalized excitations

- Possibility of the topological quantum computing with the SU(N) spin magnets??

The end